BRAINWARE UNIVERSITY

Term End Examination 2023
Programme – M.Sc.(MATH)-2022
Course Name – Complex Analysis
Course Code - MSCMC202
(Semester II)

Full Marks : 60 **Time : 2:30 Hours**

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group-A

(Multiple Choice Type Question) 1 x 15=15

1. *Choose the correct alternative from the following :*

(i) The transformation $z \to \frac{1}{z}$ transfers a straight line into a

a) Straight line passing through the origin
b) Straight line not passing through the origin
c) Circle passing through origin
d) Circle not passing through origin

(ii) By the transformation $w = cz(c > 0)$, the figure in the z -plane is stretched or contracted according as c satisfies

a) $c < 1$ or $c > 1$
b) $c > 1$ or $c < 1$
c) $c \leq 1$ or $c > 1$
d) $c \geq 1$ or $c < 1$

(iii) |z-1|=|z+i| produces

a) The disc
b) The ellipse
c) The line
d) The circle

(iv) By the transformation $w = e^{i\frac{\pi}{4}}z$, the line $x = 0$ is transformed into the line:

a) v=-u b) v=u
c) v+u=1 d) v=0

(v) Under the transformation w=z+1-i, the image of the line x=0 in the z-plane expresses

a) u=1 b) u=0
c) v=1 d) v=0

(vi) Calculate the value of $\int_C \frac{dz}{z+2}$, where $C|z| = 1$.

a) $\frac{\pi}{2}$ b) 1
c) 2πi d) 0

(vii) Solve the radius of convergence of the series $\sum_{n=0}^{\infty} \frac{n!}{n^n} z^n$.

a) 1 b) 1/e
c) e d) 0

(viii) Let $E = \{z \in C|z| > 1\} \cup \{i\}$. Then select the correct option.

a) E is open but not closed in C b) E is closed but not open in C
c) E is neither open nor closed in C d) E is both open and closed in C

(ix) Locate the region where the function $f(z) = |z|$ is analytic.

a) Everywhere b) Nowhere
c) Only at z=0 d) Everywhere except at z=0

(x) Estimate the number of isolated singularities of the function $f(z) = \frac{1}{\sin \frac{\pi}{z}}$.

a) 1 b) 2
c) 5 d) Infinite

(xi) $z = 0$ for the function $f(z) = \log z$ is categorized as

a) Isolated singularity b) Pole
c) Non-isolated singularity d) None of these

(xii) Evaluate the residue of z at $z = k\pi, k = 0, \pm 1, \pm 2, \pm 3......$

a) -1 b) 3
c) 1 d) 2.5

(xiii) Evaluate the residue of $\frac{1}{z^3 - z^5}$ at $z = 0$.

a) 1 b) 0

c) 2 d) 1.5

(xiv) Identify the region where the function sin z is analytic.

a) $C \cup \{\infty\}$ b) C except on the negative real axis

c) $C\backslash\{0\}$ d) C

(xv) Express the analytic function $w = u + iv$ where $u = e^{-x}\{(x^2 - y^2)\cos y + 2xy\sin y\}$.

a) $e^{-x}\{(x - iy)^2(\cos y - i\sin y\}$ b) $e^{-x}\{(x - iy)^2(\cos y + i\sin y\}$

c) $e^{-x}\{(x + iy)^2(\cos y - i\sin y\}$ d) $e^{-x}\{(x - iy)^2(\cos y + i\sin y\}$

Group-B

(Short Answer Type Questions) 3 x 5=15

2. Define index of curve or winding number. (3)

3. Let X=C, set of complex numbers, and define d(x+iy,a+ib)=|x-a|+|y-b|. Then show that (X,d) is a metric space. (3)

4. Examine that f(z)=xy+iy is nowhere analytic. (3)

5. Evaluate $\oint e^{-2z}dz$ from $1 - \pi i$ to $2 + 3\pi i$. (3)

6. Evaluate all the singularities of the function $f(z) = \frac{z-2}{z^2}\sin\frac{1}{z-1}$ (3)

OR

Evaluate the value of k so that the function $k\log(x^2 + y^2)$ is harmonic. (3)

Group-C

(Long Answer Type Questions) 5 x 6=30

7. State Fundamental Theorem of Algebra and prove it. (5)

8. Express $\frac{z+3}{(z-1)(z-4)}$ in Taylor series about z = 2. (5)

9. Let f be continuous on a domain D. If $\int_C f(z)dz = 0$ for every closed contour C in D, then illustrate that f is analytic throughout D. (5)

10. If f(z) is analytic and f'(z) continuous at all points inside and on simple closed curve C, then illustrate that $\oint f(z)\,dz = 0$ (5)

11. Justify-An open set G⊂C is connected iff for any two points a, b in G there is a polygon from a to b lying entirely inside G (5)

12. Justify that the transformation $w(z + i)^2 = 1$ maps the inside of the circle $|z| = 1$in z-plane on the exterior of a parabola. (5)

OR

Suppose that a function f is analytic inside and on a positively oriented circle C_R, centered at z_0 and with radius R. If M_R denotes the maximum value of |f(z)| on C_R, then justify that $\left|f^{(n)}(z_0)\right| \le \frac{n!M_R}{R^n}$ $(n = 1, 2, \ldots)$. (5)

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