BRAINWARE UNIVERSITY

Term End Examination 2023
Programme – M.Sc.(MATH)-2022
Course Name – Partial Differential Equations
Course Code - MSCMC203
(Semester II)

Full Marks : 60 **Time : 2:30 Hours**

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group-A

(Multiple Choice Type Question) 1 x 15=15

1. *Choose the correct alternative from the following :*

(i) Identify the canonical form of the differential equation $\frac{\partial^2 u}{\partial x^2} - x^2\frac{\partial^2 u}{\partial y^2} = 0$

a) $\xi = y^2 + \frac{1}{2}x, \eta = y^2 - \frac{1}{2}x$
b) $\xi = y + \frac{1}{2}x^2, \eta = y - \frac{1}{2}x^2$
c) $\xi = y + x^2, \eta = y - x^2$
d) $\xi = y^2 + x, \eta = y^2 - x$

(ii) Choose the correct option. The Laplace equation is

a) Homogeneous function
b) Non-homogeneous function
c) Harmonic function
d) None of these.

(iii) Choose the correct Lagrange's subsidiary equations for $y^2zp + zx^2q = xy^2$

a) $\frac{dx}{y^2z} = \frac{dy}{zx^2} = \frac{dz}{y^2}$
b) $\frac{dx}{x^2} = \frac{dy}{y^2} = \frac{dz}{zx}$
c) $\frac{dx}{1/x^2} = \frac{dy}{1/y^2} = \frac{dz}{1/zx}$
d) None of these

(iv) Determine the boundary condition which includes direct boundary value

a) Dirichlet Boundary Condition
b) Newmann Boundary Condition
c) Forced boundary condition
d) Discrete boundary condition

(v) Predict the general form of 2-dimensional Wave equation

a) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = k^2\frac{\partial^2 u}{\partial t^2}$
b) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{1}{k^2}\frac{\partial^2 u}{\partial t^2}$
c) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{1}{k}\frac{\partial^2 u}{\partial t^2}$
d) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{\partial^2 u}{\partial t^2}$

(vi) Identify the value of k in the heat equation $\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$

a) $k = \frac{K}{\rho\sigma}$
b) $k = \frac{K\rho}{\sigma}$
c) $k = K\rho\sigma$
d) None of these

(vii) Select the correct option. Out of the following four PDEs which equation is linear?

a) $\frac{\partial^3 z}{\partial x^3} - 3\frac{\partial^2 z}{\partial x^2}\frac{\partial z}{\partial y} + 8\frac{\partial^2 z}{\partial y^2} = \sin x$
b) $\frac{\partial^3 z}{\partial x^3} - 3\frac{\partial^2 z}{\partial x^2}\frac{\partial z}{\partial y} + 8\frac{\partial z}{\partial x}\frac{\partial^2 z}{\partial y^2} = \sin x$
c) $\frac{\partial^2 z}{\partial x^2} + 5\frac{\partial^2 z}{\partial y^2} = 0$
d) None of these

(viii) Classify the equation $\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial x \partial t} + 4\frac{\partial^2 u}{\partial x^2} = 0$

a) Elliptic
b) Hyperbolic

c) Parabolic
d) None of these

(ix) Select the correct option. The equation of the envelope of surface represented by complete integral of the given PDE is called

a) Singular solution
b) Particular Integral
c) General integral
d) None of these

(x) Choose the correct option. When solving a Laplace equation using variable separable method, we get the solution if

a) k is positive
b) k is negative
c) k is 0
d) k can be anything

(xi) The partial differential equation $z_{xx} - x^2 z_{yy} = 0, y > 0, x > 0$ can be classified as

a) Elliptic
b) Parabolic
c) Hyperbolic
d) Parabolic in $y \geq x$ and hyperbolic in $y < x$

(xii) Choose the correct option. The auxiliary equations of $p + q + 1 = 0$ are

a) $dx = dy = dz$
b) $dx = dy = -dz$
c) $\frac{dx}{p} = \frac{dy}{q} = dz$
d) $\frac{dx}{p} = \frac{dy}{q} = -dz$

(xiii) Select the PDE that refers to the governing equations of CFD…

a) Linear
b) Quasi-linear
c) Non-linear
d) Non-homogeneous

(xiv) Select the region in which the following differential equation is hyperbolic.

$$yu_{xx} + 2xyu_{xy} + xu_{yy} = u_x + u_y$$

a) $xy \neq 1$
b) $xy \neq 0$
c) $xy > 1$
d) $xy > 0$

(xv) In two-dimension heat flow, categorize the temperature along the normal to the *xy*-plane

a) zero
b) infinity
c) finite
d) 100K

Group-B

(Short Answer Type Questions) 3 x 5=15

2. Describe the characteristics of $x^2 r + 2xys + y^2 t = 0$. (3)

3. Apply the method of separating variables to solve the following problem (3)

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}, t \;\square\; 0, 0 \leq x \leq 1$$
$$u(0,t) = 2, u(1,t) = 3$$
$$u(x,0) = x(1-x)$$

4. Discuss the solution of the heat equation problem described by (3)

$$u_t(x,t) = ku_{xx}(x,t) + f(x,t), \quad -\infty < x < \infty, \qquad t > 0$$

$u(x,0) = 0, \; -\infty < x < \infty$ using Duhamel's principle.

5. Calculate the deflection $u(x,y,t)$ of the square membrane of each side unity and c=1, if the initial velocity is zero and the initial deflection is $f(x,y) = A \sin \pi x \sin 2\pi y$. (3)

6. Evaluate the complete integral of $p(1 + q) = qz$. (3)

OR

Estimate the solution of the PDE $(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$. (3)

Group-C

(Long Answer Type Questions) 5 x 6=30

7. State and prove the uniqueness theorem of the solution for the wave equation. (5)

8. Discuss the solution of the heat equation problem (5)

$$u_t(x,t) = ku_{xx}(x,t) + f(x,t), \quad -\infty < x < \infty, \quad t > 0$$

9. Deduce the two-dimensional Laplace equation in Cartesian coordinates using the method separation of variables. (5)

10. Deduce the solution of the initial value problem described by PDE: $u_{tt} - c^2\, u_{xx} = e^x$ with the given condition $u(x,0) = 5, \quad u_t(x,0) = x^2$ using any suitable solution method. (5)

11. Choose the separation of variables method and solve the following Helmholtz equation, using $\nabla^2 u + k^2 u = u_{xx} + u_{yy} + u_{zz} + k^2 u = 0$. (5)

12. Conclude that the Dirichlet problem for a rectangle is defined as (5)
PDE: $\nabla^2 u = 0,\ 0 \le x \le a, \quad 0 \le y \le b$
Boundary conditions: $u(x,b) = u(a,y) = 0, \quad u(0,y) = 0, u(x,0) = f(x)$ is of the form $u(x,y) = \sum_{n=1}^{\infty} A_n sin\, \frac{n\pi x}{a}\, sinh\, \{\frac{n\pi(y-b)}{a}\}$, where
$A_n = \frac{2}{a} \frac{1}{sinh\,(-\frac{n\pi b}{a})} \int_0^a f(x)\, sin\,(\frac{n\pi x}{a}) dx$.

OR

Decide the solution of the one-dimensional wave equation $u_{tt} = c^2 u_{xx}, \quad 0 \le x \le \pi, \quad t > 0$ subject to $i)\ u = 0\ when\ x = 0\ and\ x = \pi$
$ii)\ u_t = 0\ when\ t = 0\ and\ u(x,0) = x, \quad 0 < x < \pi$. (5)

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