BRAINWARE UNIVERSITY

Term End Examination 2023
Programme – M.Sc.(MATH)-2022
Course Name – General Topology
Course Code - MSCMC205
(Semester II)

Full Marks : 60 **Time : 2:30 Hours**

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group-A

(Multiple Choice Type Question) 1 x 15=15

1. *Choose the correct alternative from the following :*

(i) Identify the correct option: If X is first countable and Hausdorff if every convergent sequence has -

a) no limit
b) unique limit
c) finite number of limits
d) infinite number of limits

(ii) Select which of the following property is hereditary

a) Completely normal but not Kolmogorov
b) not Completely normal but Kolmogorov
c) both Completely normal and Kolmogorov
d) neither Completely normal nor Kolmogorov

(iii) Select the correct option: A space is T_5 if

a) T_1 but not completely normal
b) completely normal but not T_1
c) both T_1 and completely normal
d) neither completely normal nor T_1

(iv) Write the correct option. $\left\{1, \frac{1}{2}, \frac{1}{3}, \ldots\right\}^{\circ} = ?$

a) $[0,1]$
b) $\left\{0, 1, \frac{1}{2}, \frac{1}{3}, \ldots\right\}$
c) $\{0\}$
d) ϕ

(v) Choose the correct option: The identity function from a topological space (X, S) to (Y, T) is continuous if

a) T is finer than S
b) S is finer than T

c) T is not comparable to S | d) T=S

(vi) Write the interior set of the set Q of all rational number

a) Q | b) R
c) R\Q | d) ϕ

(vii) Let p is an accumulation point of A. If G is an open set containing p then Cite the correct one.

a) $G\cap A=\phi$ | b) $G\cap A\neq\phi$
c) $G\cap(A\setminus\{p\})=\phi$ | d) $G\cap(A\setminus\{p\})\neq\phi$

(viii) In a topological space (X, τ), the members of τ are called

a) Open sets | b) Closed sets
c) Topology members | d) None of these

(ix) Let $X=\{a,b,c\}$. If $T_1=\{X,\phi,\{a\}\}$, $T_2=\{X,\phi,\{a,b\}\}$ are two topologies on X then cite which of the following is true

a) T_1 is finer than T_2 | b) T_2 is finer than T_1
c) T_1 and T_2 are not comparable | d) $T_1=T_2$

(x) Select by which interval the lower limit topology is generated

a) (a, b) | b) [a, b)
c) (a, b] | d) [a, b]

(xi) Select the correct option: A set with same algebraic structure is called

a) Set | b) Space
c) Properset | d) Topology

(xii) Predict the correct option: Uryshon's lemma applicable on the

a) compact space | b) connected space
c) normal space | d) regular space

(xiii) Identify the correct option: If X is a Hausdorff metric space then the number of elements of the space is -

a) finite | b) countably infinite
c) uncountable | d) none of these

(xiv) Predict the correct option: Uryshon's lemma applicable on the

a) T_3 | b) T_1
c) T_4 | d) $T_{3/2}$

(xv) Write the correct option: Homeomorphism is-

a) Open map | b) Closed map
c) Structure preserving map | d) None of these

Group-B

(Short Answer Type Questions) 3 x 5=15

2. Show that R {0} is homeomorphic to xy = 1. (3)

3. Establish that the only non-empty subset of R which is open as well as closed is R. (3)

4. Examine that compactness is a topological property. (3)

5. Explain dense space (3)

6. Justify the statement: “Let f : X-Y. Then f is continuous if and only if f is sequentially continuous”. (3)

OR

Justify the statement: “Let X be a compact Hausdorff space and let A be a closed subset of X. Let $\sim$ be the equivalence relation defined on X as follows: If $x_1, x_2 \in A$ then $x_1 \sim x_2$. If $x_1, x_2 \in X A$ then $x_1 \sim x_2$ iff $x_1 = x_2$. Consider the topological space $Y := (X A) \cup \{\infty\}$ with topology Ω_Y as discussed above. Then Y is homeomorphic to $X/\sim$.” (3)

Group-C

(Long Answer Type Questions) 5 x 6=30

7. (5)

Identify a topology on X ={a,b,c} with proper figure.

8. Show that if *X* is a Hausdorff space, then a sequence of points of *X* convergesto at most one point of *X*. (5)

9. Justify that the product of two Hausdorff spaces is Hausdorff. (5)

10. Construct the proof that a circle and an ellipse are homeomorphic. (5)

11. Deduce that every compact topological space is countably compact. (5)

12. Justify that if X be a metric space with metric d. Define d’: X x X -->R by the equation d’(x , y) = min{d(x , y), 1}. Then d’ is a metric that induces the same topology as d. (5)

OR

Justify that The uniform topology on R^J is finer than the product topology and coarser than the box topology; these three topologies are all different if J is infinite (5)

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