BRAINWARE UNIVERSITY

Term End Examination 2021 - 22
Programme – Bachelor of Technology in Computer Science & Engineering
Course Name – Linear Algebra and Differential Equations
Course Code - BSC(CSE)201
(Semester II)

Time : 1 Hr.25 Min. **Full Marks : 70**

[The figure in the margin indicates full marks.]

Group-A

(Multiple Choice Type Question) 1 x 70=70

Choose the correct alternative from the following :

(1) If the matrix $\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & \lambda \end{pmatrix}$ is singular then the value of λ is

a) 3
b) 5
c) 2
d) 4

(2) If A is a non-null square matrix, then $A+A^T$ is a

a) symmetric matrix
b) skew-symmetric matrix
c) null matrix
d) none of these.

(3) $(AB)^T =$

a) A^T+B^T
b) A^TB^T
c) $B^T A^T$
d) none of these.

(4) The adjoint of the determinant $\begin{vmatrix} 2 & 1 \\ 3 & 6 \end{vmatrix}$ is

a) $\begin{vmatrix} 1 & 2 \\ 6 & 3 \end{vmatrix}$
b) $\begin{vmatrix} 6 & 3 \\ 1 & 2 \end{vmatrix}$
c) $\begin{vmatrix} -6 & 3 \\ 1 & -2 \end{vmatrix}$
d) $\begin{vmatrix} 6 & -3 \\ -1 & 2 \end{vmatrix}$

(5) If $A=\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ then $A.A^T =$

a) I_3

b) A

c) 2A

d) none of these

(6) For what value of μ does the system of equations x+y+z=1; x+2y-z=2; 5x+7y+ μz=4 have a unique solution?

a) μ≠2

b) μ≠1

c) μ≠3

d) μ≠4

(7) The system of equations x+2y-z=2, 4x+8y-4z=8 has

a) infinitely many solutions

b) no solutions

c) a unique solution

d) none of these.

(8)

The rank of the matrix $\begin{pmatrix} 2 & 2 & 1 \\ 6 & 6 & 3 \end{pmatrix}$ is

a) 2

b) 3

c) 1

d) none of these.

(9)

The value of 'k' for which rank of the matrix $\begin{pmatrix} 1 & 1 & 1 \\ 1 & k & 1 \\ 10 & 1 & 0 \end{pmatrix}$ is 2 is

a) 1

b) 0

c) -1

d) 2

(10)

If $c_1(1,0,0)+c_2(0,1,0)+c_3(0,0,1)=(0,0,1)$. Then c_1, c_2 and c_3 are respectively

a) 0,0,0

b) 0,1,0

c) 0,0,1

d) 1,1,1

(11)

$S=\{(x,y,0)\mid x,y\in R\}$ is a subspace of R^3, then dim(S) is

a) 2

b) 3

c) 5

d) None of these

(12) The expression of the vector (7,11) as a linear combination of the vectors (2,3) and (3, 5) is

a) 1(2,3)+2(3,5)

b) 2(2,3)+1(3,5)

c) cannot be expressed

d) None of these

(13)

$S=\left\{\begin{pmatrix} x_1 & 0 \\ 0 & x_2 \end{pmatrix} \mid x_1,x_2\in R\right\}$, then dim(S) is

a) 2

b) 3

c) 5

d) None of these

(14)

The vectors $(2,1,0),(1,1,0),(4,2,0)$ of R^3 are

a) linearly dependent

b) basis

c) linearly independent but not a basis
d) None of these

(15) The set $S=\{(x,2x,3x)\mid x\in R\}$ is a subspace of R^3, then dim(S) is

a) 1
b) 3
c) 4
d) None of these

(16) Let V and W be two vector spaces and $T:V\to W$ is a linear mapping, then T is injective if and only if

a) Ker T={θ}
b) Ker T={0}
c) Ker T=V
d) none

(17) If $T:V\to W$ be a linear mapping, then

a) dim (Ker T) + dim (Im T) = dim (V)
b) dim (Ker T) + dim (Im T) = dim (W)
c) dim (Ker T) + dim (Im T) = 3
d) None of these

(18) Let $T:R^3\to R^2$ be a linear transformation defined by $T(x,y,z)=(x-y,x-z)$, then the dimension of the nullspace of T is

a) 0
b) 1
c) 2
d) 3

(19) Let V be a vector space over the set of all real numbers R. Let θ be the zero vector of V. Then $2\theta=$?

a) 0
b) θ
c) 2
d) None of these

(20) Let A and B be two subspaces of a vector space V, then

a) $A\cap B$ is a subspace of V.
b) both $A\cap B$ and $A\cup B$ are subspaces of V.
c) $A\cup B$ is a subspace of V.
d) neither $A\cap B$ nor $A\cup B$ are subspaces of V.

(21) A vector space V is finite dimensional if it has

a) finite basis
b) finite elements
c) no basis
d) None of these

(22) Which one is the zero vector of R^3 under the standard vector addition?

a) (0,0)
b) (0,0,0)
c) (0,0,0,0)
d) (1,1,1)

(23) Which of the following set span the vector space $\left\{\begin{bmatrix} x & 0\\ 0 & y\end{bmatrix}: x,y\in R\right\}$?

a) $\left\{\begin{bmatrix} 1 & 0\\ 0 & 1\end{bmatrix}\right\}$
b) $\left\{\begin{bmatrix} 1 & 0\\ 0 & 1\end{bmatrix},\begin{bmatrix} 0 & 0\\ 0 & 1\end{bmatrix}\right\}$
c) $\left\{\begin{bmatrix} 1 & 0\\ 0 & 0\end{bmatrix},\begin{bmatrix} 0 & 0\\ 0 & -1\end{bmatrix}\right\}$
d) $\left\{\begin{bmatrix} 1 & 0\\ 0 & 0\end{bmatrix},\begin{bmatrix} 1 & 0\\ 0 & 1\end{bmatrix}\right\}$

(24)

Let $T: \mathbb{R}^2 \to R^3$ be a linear transformation given by $T(x,y)=(x+y, x-y, y)$, then rank of T is

a) 2
b) 0
c) 1
d) 4

(25) Let $T: V \to W$ be a liner transformation and rank(T)=m, then

a) dim(V) = m
b) dim(Ker T) = m
c) dim(Im T) = m
d) dim(W) = m

(26) Consider the mapping $M_1: T: R^3 \to R^2, T(x,y,z)=(y-1, x+z)$

$M_2: T: R^2 \to R, T(x,y)=2xy$

$M_3: T: R^3 \to R^2, T(x,y,z)=(|y|, 0)$

Which of the above is a linear transformation?

a) only M_2 and M_3
b) only M_3
c) all M_1, M_2 and M_3
d) None of these

(27) The number of vectors presents in the basis of the vector space $\left\{\begin{bmatrix} x & 0 \\ x & y \end{bmatrix} : x, y \in R\right\}$ is-

a) 1
b) 2
c) 3
d) 0

(28) If $A^2=A$, then its Eigen values are either

a) 0 or 2
b) 1 or 2
c) 0 or 1
d) Only 0

(29) If $\lambda \neq 0$ is an Eigen value of a matrix A then $\det(A-\lambda I)=$

a) λ
b) $-\lambda$
c) 2λ
d) 0

(30) The sum of the Eigen values of the matrix $A=\begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$ is

a) 5
b) -5
c) -7
d) 7

(31) If A is an orthogonal Matrix then which of the following is always true

a) $A=A^{-1}$
b) $A=-A^{-1}$
c) $A'=A^{-1}$
d) $A'=-A^{-1}$

(32) If 0 is an Eigen value of a matrix A then which of the following is false

a) 0 is an Eigen value of A^{-1}

b) 0 is an Eigen value of A^T

c) A has no inverse matrix

d) A can't be orthogonal

(33) If $\det(A) \neq 0$ then what can we say about the matrix A

a) 0 is an Eigen value of A^{-1}

b) 0 can't be an Eigen value of A^{-1}

c) $tr(A) \neq 0$

d) A is a skew-symmetric matrix

(34) If A is an orthogonal matrix then value of $\det(A)$

a) 1 or -1

b) Only 1

c) -1 or 0

d) 0 or 1

(35) If A is similar to the matrix B then

a) $A = B$

b) $A = B^{-1}$

c) $A = P^{-1}BP$

d) $A = P^T BP$

(36) If A has an Eigen vector v and $A = P^{-1}BP$ then B has an Eigen vector

a) Pv

b) $P^{-1}v$

c) v

d) v^{-1}

(37) If $V = R^3$ be equipped with inner product $(x, y) = x_1y_1 + 2x_2y_2 + 3x_3y_3$. In this inner product space $(V, (.,.))$ which of the following pairs of vectors is orthonormal?

a) $u = \begin{bmatrix}1\\0\\0\end{bmatrix}, v = \begin{bmatrix}0\\1\\0\end{bmatrix}$

b) $u = \begin{bmatrix}1\\0\\0\end{bmatrix}, v = \begin{bmatrix}0\\\frac{1}{\sqrt{2}}\\0\end{bmatrix}$

c) $u = \begin{bmatrix}\frac{1}{\sqrt{3}}\\0\\0\end{bmatrix}, v = \begin{bmatrix}0\\1\\0\end{bmatrix}$

d) $u = \begin{bmatrix}0\\0\\1\end{bmatrix}, v = \begin{bmatrix}0\\\frac{1}{2}\\0\end{bmatrix}$

(38) Consider the inner product space of all polynomial of degree less than or equal to 3 and the inner product $f(x).g(x) = \int_{-1}^{1} f(x)g(x)dx$ then the value of $x.x^3$

a) $\frac{1}{4}$

b) $\frac{1}{5}$

c) $\frac{2}{5}$

d) 0

(39)

If $\lambda = 1$ is an Eigen value of the matrix $\begin{bmatrix} \frac{3}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{3}{2} \end{bmatrix}$ then the corresponding Eigen vector is

a) $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$

b) $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

c) $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

d) $\begin{bmatrix} -1 \\ -1 \end{bmatrix}$

(40) What is the value of k so that the vectors $(1,-2,-3)$ and $(2,k,4)$ are orthogonal

a) -5

b) 5

c) -10

d) 10

(41) Which of the following matrix is orthogonally diagonalizable

a) $\begin{bmatrix} 0 & 2 & 3 \\ 2 & 0 & 4 \\ -3 & 4 & 0 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 4 \\ 3 & 4 & 3 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 2 & 3 \\ -2 & 4 & 4 \\ -3 & -4 & 3 \end{bmatrix}$

d) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 4 \\ 0 & 0 & 3 \end{bmatrix}$

(42) If the differential equation $\left(y+\frac{1}{x}+\frac{1}{x^2y}\right)dx+\left(x-\frac{1}{y}+\frac{A}{xy^2}\right)dy=0$ is exact, then the value of A is

a) 2

b) 1

c) 0

d) -1

(43) The general form of a first order linear equation in x is $\frac{dy}{dx}+Px=Q$ where

a) P and Q are both functions of x

b) P and Q are both functions of y

c) P and Q are the functions of x and y, respectively

d) P and Q are the functions of y and x, respectively

(44) The general solution of $\left(\frac{dy}{dx}\right)^2+\frac{dy}{dx}-6=0$ is

a) $(y+3x-c)(y-2x-c)=0$

b) $(y+3x-c_1)(y-2x-c_2)=0$

c) $(y+3x)(y-2x-c)=0$

d) None of these

(45) Using the substitution $x=e^z$, the equation $x^2\frac{d^2y}{dx^2}-5y=\log x$ reduces to

a) $\frac{d^2y}{dz^2}+\frac{dy}{dz}-5y=z$

b) $\frac{d^2y}{dz^2}-\frac{dy}{dz}-5y=z$

c)

d)

$\frac{d^2y}{dz^2}-2\frac{dy}{dz}-5y=z$ $\frac{d^2y}{dz^2}+\frac{dy}{dz}+5y=z$

(46) The differential equations of y and t obtained from $\frac{d^2x}{dt^2}-3x-4y=0$, $\frac{d^2y}{dt^2}+x+y=0$ is

a) $(D^4-2D^2+1)y=0$

b) $(D^4-D^2+1)y=0$

c) $(D^3-D+1)y=0$

d) None of these

(47) $\frac{1}{D^2+4}\sin 2x=$

a) $\frac{1}{4}x\cos 2x$

b) $\frac{\cos 2x}{2}$

c) $-\frac{1}{4}x\cos 2x$

d) None of these

(48) The particular integral of $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+4=e^x\cos x$ is

a) $e^x\cos x$

b) $\frac{1}{2}e^x\cos x$

c) $\frac{1}{2}e^x\sin x$

d) None of these

(49) For a given differential equation, if C.F.$=c_1\cos 2x+c_2\sin 2x$, then the Wronskian is

a) 1

b) 2

c) $\cos 2x$

d) $\sin 2x$

(50) The P.I. of the equation $2x\frac{d^2y}{dx^2}+2\frac{dy}{dx}=\frac{1}{x}$ is

a) $\frac{1}{4}x^2$

b) $\frac{1}{2}x^2$

c) $\frac{1}{2}(\log x)^2$

d) $\frac{1}{4}(\log x)^2$

(51) The differential equation involving y and t, as obtained from $\frac{dx}{dt}+2y=0$, $\frac{dy}{dt}-2y=x$ is

a) $\frac{d^2y}{dx^2}+2\frac{dy}{dt}+y=0$

b) $\frac{d^2y}{dx^2}+\frac{dy}{dt}+y=0$

c) $\frac{d^2y}{dx^2}-2\frac{dy}{dt}+y=0$

d) $\frac{d^2y}{dx^2}-\frac{dy}{dt}+y=0$

(52)

If y^a is an integrating factor of the differential equation $2xy\,dx-(3x^2-y^2)dy=0$, then the value of a is

a) -4
b) 4
c) -1
d) 1

(53) Consider two differential equations

(i) $x^2\left(\frac{d^2y}{dx^2}\right)^2+y^2\sqrt{1+\left(\frac{dy}{dx}\right)^2}=0$

(ii) $\frac{dy}{dx}-6x=\left(ay+bx\frac{dy}{dx}\right)^{\frac{3}{2}},\ b\neq 0$

The sum of the degrees of 1st and 2nd differential equation is

a) 6
b) 7
c) 8
d) 9

(54) Consider the differential equation $2\cos(y^2)dx-xy\sin(y^2)dy=0$. Then

a) e^x is an integrating factor
b) e^{-x} is an integrating factor
c) 3x is an integrating factor
d) x^3 is an integrating factor

(55) For $a,b,c\in\mathbb{R}$, if the differential equation $(ax^2+bxy+y^2)dx+(2x^2+cxy+y^2)dy=0$ is exact, then

a) a=2, c=2a
b) b=4, c=2
c) b=2, c=4
d) b=2, a=2c

(56) The C.F of the differential equation $(D^2-2D+5)^2y=\sin 2x$ is

a) $e^x\{(c_1+c_2x)\cos 2x+(c_3+c_4x)\sin 2x\}$
b) $e^x\{(c_1+c_2x)\cos x+(c_3+c_4x)\sin x\}$
c) $(c_1e^x+c_2e^{2x})\cos x\ (c_3e^x+c_4e^{2x})\sin x$
d) $(c_1\cos x+c_2\cos 2x+c_3\sin x+c_4\sin 2x)e^x$

(57) The general solution of $xp+yq=z$ for an arbitrary function ϕ is

a) $\phi\left(\frac{x}{y},xy\right)=0$
b) $\phi(x+y,xz)=0$
c) $\phi\left(\frac{x}{y},\frac{x}{z}\right)=0$
d) $\phi(xy,z)=0$

(58) The complete integral of the PDE $pq=z$, where $p=\frac{\partial z}{\partial x},\ q=\frac{\partial z}{\partial y}$ is

a) $z=(x+a)(y+b)$
b) $z=x+y+a+b$
c)
d) $z=xy+ax+by$

$z = \frac{x+a}{y+b}$

(59) The general solution of $\frac{\partial^2 z}{\partial x^2} - a^2 \frac{\partial^2 z}{\partial y^2} = 0$ is

a) $z = \phi_1(y+ax) + \phi_2(y-ax)$

b) $z = \phi_1(x+ay) + \phi_2(x-ay)$

c) $z = \phi_1(y+ax) + x\phi_2(y-ax)$

d) $z = \phi_1(x+ay) + x\phi_2(x-ay)$

(60) The particular integral of $(D^2 - 2DD' + D'^2)z = \tan(x+y)$ is

a) $\frac{x^2}{4}\tan(x+y)$

b) $\frac{x^2}{2}\cot(x+y)$

c) $\frac{x^2}{2}\tan(x+y)$

d) $\frac{x^2}{4}\cot(x+y)$

(61) The PDE $u_{xx} - 3x^2y^2u_{yy} - 2xu_x + 4yu_y = 2$ can be classified as

a) Parabola

b) Ellipse

c) Hyperbola

d) Circle

(62) Which of the following PDE represents a hyperbola?

a) $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$

b) $\frac{\partial^2 z}{\partial x^2} = k\frac{\partial z}{\partial t}$

c) $\frac{\partial^2 z}{\partial x^2} = \frac{1}{a^2}\frac{\partial^2 z}{\partial t^2}$

d) None of these

(63) The PDE $x\frac{\partial^2 z}{\partial x^2} + y\frac{\partial^2 z}{\partial y^2} = 0$ is

a) hyperbolic for $x > 0, y < 0$

b) elliptic for $x > 0, y < 0$

c) hyperbolic for $x > 0, y > 0$

d) elliptic for $x > 0, y > 0$

(64) Which of the following is parabolic?

a) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

b) $\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$

c) $\frac{\partial^2 u}{\partial t^2} = a^2\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$

d) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 3\frac{\partial^2 u}{\partial x \partial y}$

(65) When solving a 1-dimensional wave equation using variable separation method, we ge t the solution if

a) k is positive

b) k is negative

c) k is 0

d) k can be anything

(66) The PDE $r^2 + 2s - t^2 = 0$ is of order

a) 1

b) 2

c) 3

d) None of these

(67) The equation $Pp + Qq = R$ is known as

a) Lagrange's equation
b) Bernoulli's equation
c) Charpit's equation
d) Clairaut's equation

(68) The function $z - ax - by + (a^2 + b^2) = 0$ is a complete solution of

a) $z - p^2x - q^2y + (p+q) = 0$
b) $z + px + qy - (p^2 + q^2) = 0$
c) $z - px + qy - (p^2 + q^2) = 0$
d) $z - px - qy + (p^2 + q^2) = 0$

(69) A partial differential equation has

a) one independent variable
b) two or more independent variables
c) more than one dependent variables
d) equal number of dependent and independent variables

(70) If $f(x,y,z) = x^2 + xyz + z$ find f_x at (1, 1, 1)

a) 0
b) 1
c) 3
d) -1