BRAINWARE UNIVERSITY

Term End Examination 2023-2024
Programme – B.Tech.(CSE)-DS-2022/B.Tech.(CSE)-DS-2023
Course Name – Calculus & Linear Algebra
Course Code - BSCD102
(Semester I)

Full Marks : 60 **Time : 2:30 Hours**

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group-A
(Multiple Choice Type Question) 1 x 15=15

1. *Choose the correct alternative from the following :*

(i) Choose eigenvalues of a 3x3 diagonal matrix are given by

a) The elements on the main diagonal
b) The sum of the elements on the main diagonal
c) The product of the elements on the main diagonal
d) The inverse of the elements on the main diagonal

(ii) Select the correct value of $\Gamma(1)$

a) π
b) 1
c) $\frac{\sqrt{\pi}}{2}$
d) $\frac{1}{2}$

(iii) Choose eigenvalues of a 2x2 rotation matrix are:

a) 1 and 1
b) 1 and -1
c) 0 and 1
d) Complex numbers in the form cos(θ) ± i sin(θ)

(iv) If a matrix has all its eigenvalues equal to 1,t establish that

a) The matrix is singular
b) The matrix is defective
c) The matrix is orthogonal
d) The matrix is an identity matrix

(v) Choose the correct determinant of a 1x1 matrix [a] from following given options

a) a
b) 1
c) 0
d) -a

(vi) Choose the correct option. The sequence $\{(-1)^n . 2^n\}$ is

a) monotone
b) bounded
c) convergent
d) oscillatory infinitely

(vii) Examine the type of the series $1 + 2 + 3 + \ldots$

a) Convergent
b) Divergent

c) Absolutely convergent
d) None of these

(viii) If $f(x,y) = 0$, then calculate $\frac{dy}{dx} =$

a) $\frac{f_x}{f_y}$
b) $\frac{f_y}{f_x}$
c) $-\frac{f_x}{f_y}$
d) $-\frac{f_y}{f_x}$

(ix) Identify the value of 'a' for which rank of the matrix $\begin{pmatrix} 2 & 0 & 1 \\ 5 & a & 3 \\ 0 & 3 & 1 \end{pmatrix}$ is less than 3.

a) $\frac{3}{4}$
b) $\frac{3}{5}$
c) $\frac{3}{2}$
d) 1

(x) Choose the correct option The rank of a matrix cannot exceed:

a) The number of rows
b) The number of columns
c) The minimum of the number of rows and columns
d) The maximum of the number of rows and columns

(xi) If the rank of a 3x3 matrix is 3 then choose the correct option

a) The matrix is singular
b) The matrix is non invertible
c) The matrix has a determinant of zero
d) The matrix has three linearly independent rows and columns

(xii) Identify when a matrix has a rank 0

a) When it has all elements equal to 1
b) When it is an identity matrix
c) When it has no non-zero elements
d) When it has only one row or one column

(xiii) Select the value of $\Gamma\left(\frac{1}{3}\right)\Gamma\left(\frac{2}{3}\right)$ is

a) $\frac{2\pi}{\sqrt{3}}$
b) $\frac{3\pi}{\sqrt{2}}$
c) $\frac{\pi}{\sqrt{3}}$
d) $\frac{\pi}{\sqrt{2}}$

(xiv) Compute $\int_0^\infty e^{-x^2} dx =$

a) π
b) $\sqrt{\pi}$
c) $\frac{\sqrt{\pi}}{2}$
d) $\frac{\pi}{2}$

(xv) For $k > 0, n > 0$, Evaluate $\int_0^\infty e^{-kt} t^{n-1} dt =$

a) $\frac{\Gamma(n)}{k^n}$
b) $\frac{\Gamma(k)}{k^n}$
c) $\frac{\Gamma(k)}{n^n}$
d) $\frac{\Gamma(k)}{k}$

Group-B

(Short Answer Type Questions) 3 x 5=15

2. Determine whether the set of vectors $\{(a, b) \in \mathbb{R}^2 : 12b = 7a + 5\}$ is a vector space. (3)

3. Explain Lagrange's theorem. (3)

4. Examine the convergence of the series $\sum_{n=1}^{\infty} e^{-n} n!$ (3)

5. Calculate the value of $\int_{-\infty}^{0} \frac{1}{x^2+4} dx$. (3)

6. Without expansion, illustrate that $\begin{vmatrix} 6 & 1 & 3 & 2 \\ -2 & 0 & 1 & 4 \\ 3 & 6 & 1 & 2 \\ -4 & 0 & 2 & 8 \end{vmatrix} = 0$. (3)

OR

Calculate the inverse, if it exists, of the matrix $\begin{pmatrix} 0 & -2 & -3 \\ 1 & 3 & 3 \\ -1 & -2 & -2 \end{pmatrix}$. (3)

Group-C
(Long Answer Type Questions) 5 x 6=30

7. Establish that $\int_0^{\frac{\pi}{2}} \sin^4 x \cos^4 x \ dx = \frac{3\pi}{256}$. (5)

8. Establish that $\lim_{n\to\infty} \left(\frac{1}{\sqrt{n^2+1}} + \frac{1}{\sqrt{n^2+2}} + \cdots + \frac{1}{\sqrt{n^2+n}}\right) = 1$ (5)

9. Determine whether the given matrix A is diagonalizable. If so, find the matrix P that diagonalizes $A=\begin{pmatrix} -9 & 13 \\ -2 & 6 \end{pmatrix}$ and the diagonal matrix D such that $D = P^{-1}AP$. (5)

10. Illustrate that for the function $f(x,y) = \frac{x^2y^2}{x^2+y^2}$, $(x,y) \neq (0,0)$
$= 0, \quad (x,y) = (0,0)$ (5)

$$f_{xy}(0,0) = f_{yx}(0,0)$$

11. If $\det\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} = 5$, evaluate the determinant of the matrix $\begin{bmatrix} 2a_1 & a_2 & a_3 \\ 6b_1 & 3b_2 & 3b_3 \\ 2c_1 & c_2 & c_3 \end{bmatrix}$. (5)

12. Evaluate the eigenvalues and eigenvectors of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 5 & 6 \\ 0 & 0 & -7 \end{bmatrix}$. (5)

OR (5)

Deduce the set of vectors $v_1 = (1,0,0), v_2 = (0,1,-1), v_3 = (0,4,-3), v_4 = (0,2,0)$ to obtain a basis of $\mathbb{R}^3$.

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