BRAINWARE UNIVERSITY

Term End Examination 2023-2024

Programme – B.Tech.(CSE)-AIML-2021/B.Tech.(CSE)-AIML-2022/B.Tech.(CSE)-AIML-2023

Course Name – Calculus & Linear Algebra

Course Code - BSCM102

(Semester I)

Full Marks : 60 **Time : 2:30 Hours**

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group-A

(Multiple Choice Type Question) 1 x 15=15

1. *Choose the correct alternative from the following :*

(i) In a 3-dimensional vector space, choose the maximum number of linearly independent vectors that can exist

a) 1
b) 2
c) 3
d) 4

(ii) Identify the value of 'a' for which rank of the matrix $\begin{pmatrix} 2 & 0 & 1 \\ 5 & a & 3 \\ 0 & 3 & 1 \end{pmatrix}$ is less than 3.

a) $\frac{3}{4}$
b) $\frac{3}{5}$
c) $\frac{3}{2}$
d) 1

(iii) $S = \{(x, y, 0) | x, y \in R\}$ is a subspace of R^3, then determine dim(S) is

a) 2
b) 3
c) 5
d) None of these

(iv) Choose the correct option

a) The rank of a matrix is number of rows in the matrix
b) The rank of a matrix is number of columns in the matrix
c) The rank of a matrix is maximum value in the matrix
d) The rank of a matrix is maximum number of linearly independent rows or columns in the matrix

(v) If $f(x)$ satisfy all the conditions of Rolle's theorem in $[a, b]$, then identify where $f'(x)$ becomes zero

a) only at one point in (a, b)
b) at two points in (a,b)

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c) at least one point in (a,b)

d) none of these

(vi) Select the value of $\Gamma\left(\frac{1}{3}\right)\Gamma\left(\frac{2}{3}\right)$ is

a) $\frac{2\pi}{\sqrt{3}}$

b) $\frac{3\pi}{\sqrt{2}}$

c) $\frac{\pi}{\sqrt{3}}$

d) $\frac{\pi}{\sqrt{2}}$

(vii) For all second-degree polynomials with $y = ax^2 + bx + k$, it is seen that the Rolles' point is at $c = 0$. Also, the value of k is zero. Choose the correct value of b.

a) 0

b) 1

c) -1

d) 36

(viii) Compute $\int_{-\pi}^{\pi} \sin 6x dx =$

a) 0

b) 1

c) -1

d) None of these

(ix) Examine the convergence of the sequence $\left\{1, \frac{1}{3}, \frac{1}{3^2}, \ldots\ldots, \frac{1}{3^n}, \ldots. \infty\right\}$

a) convergent

b) divergent

c) oscillatory

d) none of these

(x) Choose the correct option: If two rows of a matrix are proportional, the determinant is:

a) 0

b) 1

c) -1

d) Not defined

(xi) Examine the type of the series $1 + 2 + 3 + \ldots$

a) Convergent

b) Divergent

c) Absolutely convergent

d) None of these

(xii) If $f(x,y) = x^2 + y^2$ then compute $f_{xy}(x,y) =$

a) 1

b) 0

c) 2

d) x+y

(xiii) Choose the correct option: If the dimension of a vector space is n, then any set of more than n vectors is:

a) Linearly dependent

b) Linearly independent

c) Perpendicular to each other

d) Orthogonal to the zero vector

(xiv)

Choose the correct option

The intersection of two subspaces of a vector space is always:

a) The zero vector.

b) A single vector.

c) A subspace.

d) Empty

(xv) Write which of the following properties must be satisfied for a set of vectors to form a vector space?

a) Closure under vector addition and scalar multiplication.

b) Closure under matrix multiplication.

c) Contains only a single vector.

d) Contains only unit vectors.

Group-B

(Short Answer Type Questions) 3 x 5=15

2. Calculate the value of $\int_{-\infty}^{0} \frac{1}{x^2+4} dx$. (3)

3. Determine whether the set of vectors $\{(a, b) \in \mathbb{R}^2 : b = 3a + 1\}$ is a vector space. (3)

4. Show that $\frac{(b-a)}{\sqrt{1-a^2}} < \sin^{-1} b - \sin^{-1} a < \frac{(b-a)}{\sqrt{1-b^2}}$ if $0 < a < b < 1$ (3)

5. Examine the convergence of the series $\sum_{n=1}^{\infty} e^{-n} n!$ (3)

6. If x= - 4 is a root of $\begin{vmatrix} x & 2 & 3 \\ 1 & x & 1 \\ 3 & 2 & x \end{vmatrix} = 0$, calculate the other roots (3)

OR

If $\begin{vmatrix} 4-x & 4+x & 4+x \\ 4+x & 4-x & 4+x \\ 4+x & 4+x & 4-x \end{vmatrix} = 0$ then calculate values of x. (3)

Group-C

(Long Answer Type Questions) 5 x 6=30

7. Given B = {u, v}, where u = (0, 1) and v = (1, 1), use the Gram-Schmidt procedure to evaluate a corresponding orthonormal basis. (5)

8. Calculate the eigenvalues and eigenvectors of matrix A=$\begin{pmatrix} 5 & 4 \\ 1 & 2 \end{pmatrix}$. (5)

9. Establish that $\int_{0}^{\frac{\pi}{2}} \sin^4 x \cos^4 x \; dx = \frac{3\pi}{256}$. (5)

10. Calculate the extrema of the following function: (5)

$$f(x, y) = x^3 + 3xy^2 - 3y^2 - 3x^2 + 4$$

11. Calculate the interval and radius of convergence $\sum n!.x^n$. (5)

12. A mapping $T:\mathbb{R}^3 \rightarrow \mathbb{R}^3$ is defined by $T(x_1, x_2, x_3) = (x_1 + x_2 + x_3, 2 + x_2 + 2x_3, x_1 + 2x_2 + x_3), (x_1, x_2, x_3) \in \mathbb{R}^3$. Conclude that T is a linear mapping. (5)

OR

Evaluate the kernel of the mapping $T:\mathbb{R}^3 \rightarrow \mathbb{R}^2$ is defined by $T(x_1, x_2, x_3) = (x_1 + x_2, x_2 - x_3)$. (5)

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