BRAINWARE UNIVERSITY

Term End Examination 2023-2024
Programme – M.Tech.(CSE)-AIML-2022/M.Tech.(CSE)-AIML-2023
Course Name – Mathematics -I
Course Code - BSC-MMM101
(Semester I)

Full Marks : 60 **Time : 2:30 Hours**

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group-A
(Multiple Choice Type Question) 1 x 15=15

1. *Choose the correct alternative from the following :*

(i) For an irreducible, positive recurrent, time homogeneous continuous-time Markov Chain, consider the following statements

[1] Limiting distribution does not exist

[2] Stationary distribution exists

[3] Limiting distribution and stationary distribution both exist and are same.

Select the correct option based on the above three statements:

a) All three statements are true.
b) Statement 1 is always true but 2 and 3 may or may not be true.
c) Statement 2 and 3 are always true but 1 is not true.
d) None of the statements are necessarily true.

(ii) If $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{4}$, $P(A \cup B) = \frac{1}{2}$, then compute $P(B|A)$.

a) $\frac{3}{4}$
b) $\frac{4}{3}$
c) $\frac{1}{4}$
d) $\frac{1}{3}$

(iii) If the coefficients $\{\lambda_j\}$and $\{\mu_j\}$are called the birth and death rates respectively. Then a process is said to be pure death process if_____. Select the correct option.

a) $\mu_j = 0$
b) $\mu_j = 1$
c) $\lambda_j = 0$
d) $\lambda_j = 1$

(iv) The steady-state probability vector π of a discrete Markov chain with transition probability matrix P satisfies the matrix equation of _____. Select the correct option.

a) $P\pi = 0$
b) $(1 - P)\pi = 0$
c) $P\pi = \pi$
d) $P^t\pi = 0$

(v) If the coefficients $\{\lambda_j\}$and $\{\mu_j\}$are called the birth and death rates respectively. Then a process is said to be pure birth process if _____. Select the correct option.

a) $\mu_j = 0$ b) $\mu_j = 1$

c) $\lambda_j = 0$ d) $\lambda_j = 1$

(vi) A transition probability matrix P is said to be doubly stochastic if_____. Select the correct option.

a) the column sums all equal to 1 b) the row sums all equal to 1

c) the column sums all less than 1 d) the roe sums all less than 1

(vii) Let X be an N0-valued random variable, and let P(s) be its generating function, and let Q(s) = P(s)/(1 - s). Then select which of the following alternative is true.

a) Q(s) is a generating function of a random variable, b) Q(s) is a generating function of a non-decreasing sequence of non-negative numbers,

c) Q(s) is a concave function on (0,1) d) Q(0) = 1,

(viii) In a Markov chain whose state space has n elements (n belongs to set of natural numbers), _____. Select the correct option.

a) all classes are closed b) at least one state is transient,

c) not more than half of all states are transient, d) there are at most n classes,

(ix) Compute the variance of first n natural numbers.

a) n^2-1 b) $(n^2-1)/10$

c) $n^2/12$ d) $(n^2-1)/12$

(x) If P(A)=1/2,P(B)=1/3, and P(A∩B)=1/4, then compute the value of P(A∪B).

a) 6/7 b) 3/7

c) 1 d) 7/12

(xi) Service mechanism in a queuing system is characterized by _____. Select the correct option.

a) Server's behaviour b) Customer behaviour

c) Customers in the system d) none of the these

(xii) Sum of total probabilities in a sample space is _____. Select the correct option

a) 1/2 b) 1

c) 0 d) Cann't say

(xiii) Number of times each value appears is called value\'s _____. Select the correct option.

a) range b) frequency

c) mode d) standard deviation

(xiv) The mean of the binomial distribution is _____. Select the correct option.

a) less than the variance b) equal to its variance

c) greater than its variance d) greater than or equal to its variance

(xv) A set of all possible outcomes of an experiment is called _____. Select the correct option.

a) Combination b) Sample point

c) Sample space d) Compound event

Group-B

(Short Answer Type Questions) 3 x 5=15

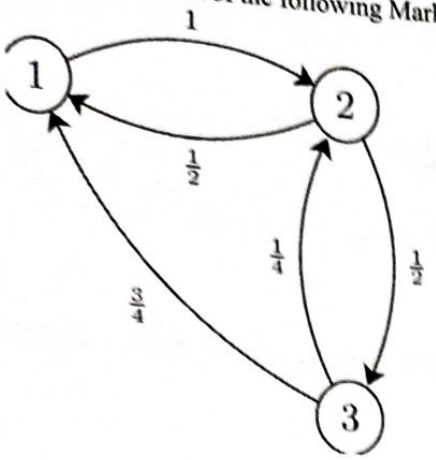
2. Define mean and variance of continuous random variable. (3)

3. Explain the concept of expectation of continuous random variable with an example. (3)

4. Define stochastic processes. (3)

5. (3)

Evaluate the TPM of the following Markov chain shown in Figure:



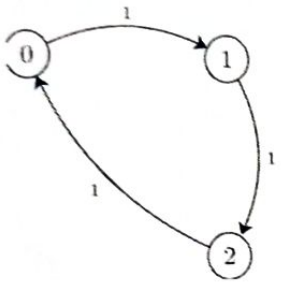
6. Consider the following transition matrix (3)

$$\begin{bmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

Justify if the matrix is valid TPM.

OR (3)

Evaluate the TPM of the following Markov chain shown in Figure:



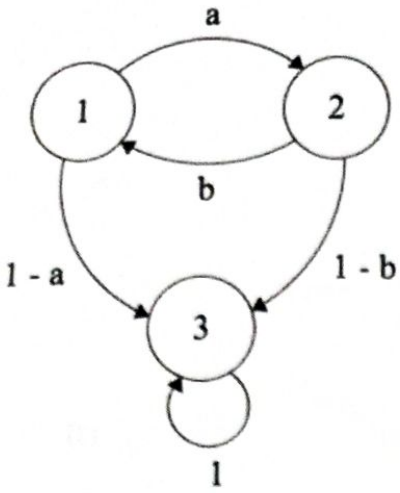
Group-C

(Long Answer Type Questions) 5 x 6=30

7. Explain Bayes' theorem. (5)

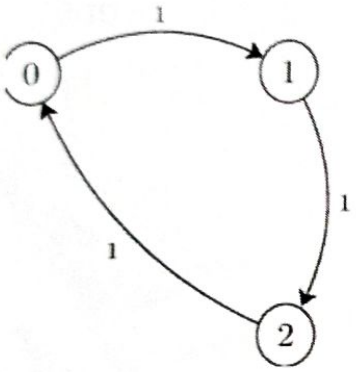
8. An HIV test gives a positive result with probability 98% when the patient is indeed affected by HIV, while it gives a negative result with 99% probability when the patient is not affected by HIV. If a patient is drawn at random from a population in which 0.1% of individuals are affected by HIV and he is found positive, compute the probability that he is indeed affected by HIV? (5)

9. Consider the Markov chain shown in Figure (5)



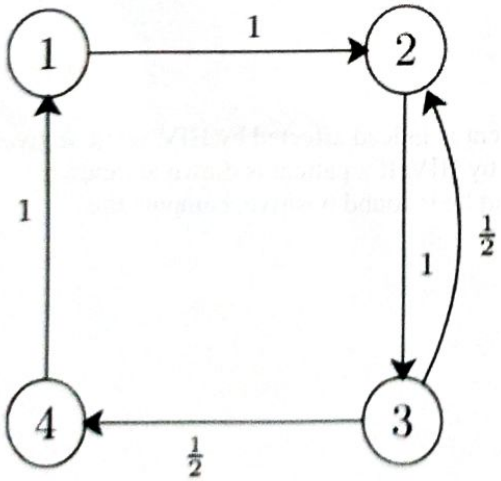
Choose a and b so that it consists a valid TPM.

10. Consider the Markov chain shown in Figure (5)



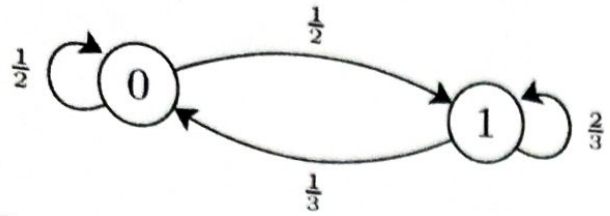
Evaluate the stationary probabilities of the chain.

11. Consider the Markov chain shown in Figure (5)



Evaluate the stationary probabilities of the chain.

12. Consider the Markov chain shown in Figure (5)



Evaluate the stationary probabilities of the chain.

OR

Consider the Markov chain with two states and transition probability matrix $P = \begin{bmatrix} \frac{3}{4} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$. (5)
Evaluate the stationary probabilities of the chain.

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