BRAINWARE UNIVERSITY

Term End Examination 2023-2024
Programme – B.Tech.(RA)-2023
Course Name – Calculus & Linear Algebra
Course Code - BSCR102
(Semester I)

Full Marks : 60

Time : 2:30 Hours

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group-A
(Multiple Choice Type Question)   1 x 15=15

1. Choose the correct alternative from the following :

(i) Choose the correct statement about the eigenvalues of a unitary matrix?
   a) All eigenvalues have a magnitude of 1
   b) All eigenvalues are real numbers
   c) All eigenvalues are distinct
   d) All eigenvalues are negative numbers

(ii) Choose the correct determinant of a 1x1 matrix [a] from following given options
   a) a
   b) 1
   c) 0
   d) -a

(iii) Examine the convergence of the sequence $\{x_n\}$, where $x_n = (-1)^{n-1}$, is a
   a) Convergent sequence
   b) Divergent sequence
   c) Oscillating sequence
   d) None of these

(iv) Choose the correct option, the sequence $\left\{\frac{1}{n^p}\right\}$, where $p > 0$ is
   a) Null sequence
   b) Divergent sequence
   c) Constant sequence
   d) None of these

(v) If $f(x, y) = 0$, then calculate $\frac{dy}{dx} =$
   a) $\frac{f_x}{f_y}$
   b) $\frac{f_y}{f_x}$
   c) $-\frac{f_x}{f_y}$
   d) $-\frac{f_y}{f_x}$

(vi) If $c_1(1,0,0) + c_2(0,1,0) + c_3(0,0,1) = (0,0,1)$. Then determine c_1, c_2 and c_3 are respectively
   a) 0, 0, 0
   b) 0, 1, 0
   c) 0, 0, 1
   d) 1, 1, 1

(vii) $S = \{(x, y, 0)|x, y \in R\}$ is a subspace of R^3, then determine dim(S) is
   a) 2
   b) 3

c) 5

d) None of these

(viii) If A is an orthogonal Matrix then identify the correct option?

a) $A = A^{-1}$

b) $A = -A^{-1}$

c) $A^T = A^{-1}$

d) $A^T = -A^{-1}$

(ix) Select the rank of the zero matrix

a) 0

b) 1

c) Depends on the size of the matrix

d) Cannot be determined

(x) If $f(x)$ satisfy all the conditions of Rolle's theorem in $[a, b]$, then identify where $f'(x)$ becomes zero

a) only at one point in (a, b)

b) at two points in (a,b)

c) at least one point in (a,b)

d) none of these

(xi) Compute $\int_0^\infty e^{-x^2} dx =$

a) π

b) $\sqrt{\pi}$

c) $\frac{\sqrt{\pi}}{2}$

d) $\frac{\pi}{2}$

(xii) For $k > 0, n > 0$, Evaluate $\int_0^\infty e^{-kt} t^{n-1} dt =$

a) $\frac{\Gamma(n)}{k^n}$

b) $\frac{\Gamma(k)}{k^n}$

c) $\frac{\Gamma(k)}{n^n}$

d) $\frac{\Gamma(k)}{k}$

(xiii) Choose eigenvalues of a 3x3 diagonal matrix are given by:

a) The elements on the main diagonal

b) The sum of the elements on the main diagonal

c) The product of the elements on the main diagonal

d) The inverse of the elements on the main diagonal

(xiv) Select the value of $\lim\limits_{x \to 0+} x^x$

a) 1

b) 0

c) 2

d) 3

(xv) Choose the correct option: If a matrix A is diagonalizable, it means:

a) A can be written in the form $A = \lambda v$

b) A can be written in the form $A = PDP^{-1}$

c) A can be written in the form $A = \lambda I$

d) A can be written in the form $A = P - D$

Group-B
(Short Answer Type Questions)

3 x 5=15

2. Examine the convergence of the series $\sum_{n=1}^\infty \frac{1}{n^2+5n}$ (3)

3. Calculate the value of $\sum_{n=1}^\infty (-1)^{n+1} \frac{3}{2^n}$ (3)

4. Establish that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$. (3)

5. Determine whether the vectors $v_1 = (1, 2, 3)$, $v_2 = (1, 0, 1)$, $v_3 = (1, -1, 5)$ in $\mathbb{R}^3$ is linearly dependent or linearly independent. (3)

6.

Without expanding illustrate that $\begin{vmatrix} 0 & b-a & c-a \\ a-b & o & c-b \\ a-c & b-c & 0 \end{vmatrix} = 0.$

<div align="center">OR</div>

(3)

If a+b+c $\neq$ 0 and $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$ then illustrate that a = b = c.

Group-C
(Long Answer Type Questions)

5 x 6=30

7.
   Illustrate $\int_0^{\frac{\pi}{2}} \sqrt{tanx}\ dx = \frac{\pi}{\sqrt{2}}.$

(5)

8.
   Given $A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}$ use elementary row operations on A to calculate A^{-1}

(5)

9. Examine whether the limit of $f(x,y) = \frac{xy}{y^2-x^2}$ exist when $(x,y) \to (0,0)$.

(5)

10.
   Calculate the interval and radius of convergence $\sum \frac{(x-2)^n}{(n+1)n^3}.$

(5)

11.
   Given B = {u, v, w}, where u = (1, 2, 1), v = (1, 1, 3) and w = (2, 1, 1), use the Gram-Schmidt procedure to evaluate a corresponding orthonormal basis.

(5)

12. A mapping $T: \mathbb{R}^3 \to \mathbb{R}^3$ is defined by $T(x_1, x_2, x_3) = (x_1 + x_2 + x_3, 2 + x_2 + 2x_3, x_1 + 2x_2 + x_3), (x_1, x_2, x_3) \in \mathbb{R}^3$. Conclude that T is a linear mapping.

(5)

<div align="center">OR</div>

A linear mapping $T: R^3 \to R^4$ is defined by $T(x_1, x_2, x_3) = (x_2 + x_3, x_3 + x_1, x_1 + x_2, x_1 + x_2 + x_3), (x_1, x_2, x_3) \in R^3$. Justify that T is a linear mapping. Determine $Ker\ T$.

(5)

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