



BRAINWARE UNIVERSITY

Term End Examination 2023-2024

Programme – B.Tech.(RA)-2023

Course Name – Calculus & Linear Algebra

Course Code - BSCR102

(Semester I)

Library
303, Brainware University
Ranikrishnapur Road, Barasat
Kolkata, West Bengal-700125

Full Marks : 60

Time : 2:30 Hours

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group-A

(Multiple Choice Type Question)

1 x 15=15

1. Choose the correct alternative from the following :

- (i) Choose the correct statement about the eigenvalues of a unitary matrix?
 - a) All eigenvalues have a magnitude of 1
 - b) All eigenvalues are real numbers
 - c) All eigenvalues are distinct
 - d) All eigenvalues are negative numbers
- (ii) Choose the correct determinant of a 1×1 matrix [a] from following given options
 - a) a
 - b) 1
 - c) 0
 - d) -a
- (iii) Examine the convergence of the sequence $\{x_n\}$, where $x_n = (-1)^{n-1}$, is a
 - a) Convergent sequence
 - b) Divergent sequence
 - c) Oscillating sequence
 - d) None of these
- (iv) Choose the correct option, the sequence $\left\{\frac{1}{n^p}\right\}$, where $p > 0$ is
 - a) Null sequence
 - b) Divergent sequence
 - c) Constant sequence
 - d) None of these
- (v) If $f(x, y) = 0$, then calculate $\frac{dy}{dx} =$
 - a) $\frac{f_x}{f_y}$
 - b) $\frac{f_y}{f_x}$
 - c) $-\frac{f_x}{f_y}$
 - d) $-\frac{f_y}{f_x}$
- (vi) If $c_1(1,0,0) + c_2(0,1,0) + c_3(0,0,1) = (0,0,1)$. Then determine c_1 , c_2 and c_3 are respectively
 - a) 0, 0, 0
 - b) 0, 1, 0
 - c) 0, 0, 1
 - d) 1, 1, 1
- (vii) $S = \{(x, y, 0) | x, y \in R\}$ is a subspace of R^3 , then determine $\dim(S)$ is
 - a) 2
 - b) 3

- (viii) If A is an orthogonal Matrix then identify the correct option?
- a) $A = A^{-1}$
 - b) $A = -A^{-1}$
 - c) $A^T = A^{-1}$
 - d) None of these
- (ix) Select the rank of the zero matrix
- a) 0
 - b) 1
 - c) Depends on the size of the matrix
 - d) Cannot be determined
- (x) If $f(x)$ satisfy all the conditions of Rolle's theorem in $[a, b]$, then identify where $f'(x)$ becomes zero
- a) only at one point in (a, b)
 - b) at two points in (a, b)
 - c) at least one point in (a, b)
 - d) none of these
- (xi) Compute $\int_0^\infty e^{-x^2} dx =$
- a) π
 - b) $\sqrt{\pi}$
 - c) $\frac{\sqrt{\pi}}{2}$
 - d) $\frac{\pi}{2}$
- (xii) For $k > 0, n > 0$, Evaluate $\int_0^\infty e^{-kt} t^{n-1} dt =$
- a) $\frac{\Gamma(n)}{k^n}$
 - b) $\frac{\Gamma(k)}{k^n}$
 - c) $\frac{\Gamma(k)}{n^n}$
 - d) $\frac{\Gamma(k)}{k}$
- (xiii) Choose eigenvalues of a 3×3 diagonal matrix are given by:
- a) The elements on the main diagonal
 - b) The sum of the elements on the main diagonal
 - c) The product of the elements on the main diagonal
 - d) The inverse of the elements on the main diagonal
- (xiv) Select the value of $\lim_{x \rightarrow 0^+} x^x$
- a) 1
 - b) 0
 - c) 2
 - d) 3
- (xv) Choose the correct option: If a matrix A is diagonalizable, it means:
- a) A can be written in the form $A = \lambda v$
 - b) A can be written in the form $A = PDP^{-1}$
 - c) A can be written in the form $A = \lambda I$
 - d) A can be written in the form $A = P - D$
- Group-B**
(Short Answer Type Questions) 3 x 5=15
2. Examine the convergence of the series $\sum_{n=1}^{\infty} \frac{1}{n^2 + 5n}$ (3)
3. Calculate the value of $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{3}{2^n}$ (3)
4. Establish that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$. (3)
5. Determine whether the vectors $v_1 = (1, 2, 3)$, $v_2 = (1, 0, 1)$, $v_3 = (1, -1, 5)$ in \mathbb{R}^3 is linearly dependent or linearly independent. (3)

6.

Without expanding illustrate that $\begin{vmatrix} 0 & b-a & c-a \\ a-b & 0 & c-b \\ a-c & b-c & 0 \end{vmatrix} = 0$.

(3)

OR

(3)

If $a+b+c \neq 0$ and $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$ then illustrate that $a = b = c$.

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Group-C
(Long Answer Type Questions)

5 x 6=30

7.

Illustrate $\int_0^{\frac{\pi}{2}} \sqrt{\tan x} dx = \frac{\pi}{\sqrt{2}}$. (5)

8.

Given $A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}$ use elementary row operations on A to calculate A^{-1} (5)

9. Examine whether the limit of $f(x, y) = \frac{xy}{y^2-x^2}$ exist when $(x, y) \rightarrow (0, 0)$. (5)

10.

Calculate the interval and radius of convergence $\sum \frac{(x-2)^n}{(n+1)n^3}$. (5)

11.

Given $B = \{u, v, w\}$, where $u = (1, 2, 1)$, $v = (1, 1, 3)$ and $w = (2, 1, 1)$, use the Gram-Schmidt procedure to evaluate a corresponding orthonormal basis. (5)

12. A mapping $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ is defined by $T(x_1, x_2, x_3) = (x_1 + x_2 + x_3, 2 + x_2 + 2x_3, x_1 + 2x_2 + x_3)$, $(x_1, x_2, x_3) \in \mathbb{R}^3$. Conclude that T is a linear mapping. (5)

OR

A linear mapping $T: \mathbb{R}^3 \rightarrow \mathbb{R}^4$ is defined by $T(x_1, x_2, x_3) = (x_2 + x_3, x_3 + x_1, x_1 + x_2, x_1 + x_2 + x_3)$, $(x_1, x_2, x_3) \in \mathbb{R}^3$. Justify that T is a linear mapping. Determine $\text{Ker } T$. (5)

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Document

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Date: 2023-09-15

Subject: Application for admission to Undergraduate Program in Computer Science and Engineering (B.Tech)

Dear Sir/Madam, I am writing to you to apply for admission to your institution.

I am currently a final year student of [Your Current School/College] in [Your Current Course]. I have completed my studies with a CGPA of [Your CGPA] and am looking forward to continuing my education at your esteemed institution.

I am particularly interested in the Bachelor of Technology (B.Tech) program in Computer Science and Engineering offered by your university. This program aligns well with my academic interests and career goals.

I have a strong academic background in Mathematics, Physics, Chemistry, and English. I also have experience in programming languages such as C, C++, Java, and Python, along with a basic understanding of databases and operating systems.

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