BRAINWARE UNIVERSITY

Term End Examination 2023-2024
Programme – B.Tech.(CSE)-2023
Course Name – Calculus & Linear Algebra
Course Code - BSCG102
(Semester I)

Full Marks : 60 **Time : 2:30 Hours**

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group-A
(Multiple Choice Type Question) 1 x 15=15

1. *Choose the correct alternative from the following :*

(i) Identify the value of 'a' for which rank of the matrix $\begin{pmatrix} 2 & 0 & 1 \\ 5 & a & 3 \\ 0 & 3 & 1 \end{pmatrix}$ is less than 3.

a) $\frac{3}{4}$ b) $\frac{3}{5}$
c) $\frac{3}{2}$ d) 1

(ii) Select the rank of the zero matrix

a) 0 b) 1
c) Depends on the size of the matrix d) Cannot be determined

(iii) If $f(x)$ satisfy all the conditions of Rolle's theorem in $[a, b]$, then identify where $f'(x)$ becomes zero

a) only at one point in (a, b) b) at two points in (a,b)
c) at least one point in (a,b) d) none of these

(iv) For two matrices A and B of the same size then choose the correct statement

a) If rank(A) = rank(B), then A = B b) If rank(A) < rank(B), then A = B
c) If rank(A) > rank(B), then A = B d) None of these

(v) Select the correct value of $\beta\left(\frac{1}{2}, \frac{1}{2}\right)$ is

a) π b) $\sqrt{\pi}$
c) $\frac{\sqrt{\pi}}{2}$ d) $\frac{\pi}{2}$

(vi) Choose the correct option If the rank of a matrix is less than the number of rows then the matrix is

118 - 488

a) Non-singular
b) Singular
c) Square
d) Symmetric

(vii) Compute $\int_0^{\infty} e^{-x^2}dx =$

a) π
b) $\sqrt{\pi}$
c) $\frac{\sqrt{\pi}}{2}$
d) $\frac{\pi}{2}$

(viii) For $k > 0, n > 0$, Evaluate $\int_0^{\infty} e^{-kt}t^{n-1}dt =$

a) $\frac{\Gamma(n)}{k^n}$
b) $\frac{\Gamma(k)}{k^n}$
c) $\frac{\Gamma(k)}{n^n}$
d) $\frac{\Gamma(k)}{k}$

(ix) Choose matrix whose all its eigenvalues equal to zero is called:

a) Nilpotent matrix
b) Diagonal matrix
c) Identity matrix
d) Invertible matrix

(x) Choose the correct option

The eigenvalues of a Hermitian matrix are always:

a) Real numbers
b) Complex numbers with a positive real part
c) Complex numbers with a negative real part
d) Imaginary numbers

(xi) Choose the correct option The eigenvectors of a matrix A form a basis for:

a) The null space of A
b) The column space of A
c) The row space of A
d) The vector space on which A operates

(xii) Choose the correct option For a positive definite matrix, the eigenvalues are:

a) All negative
b) All positive
c) All zero
d) A mix of positive and negative

(xiii) Examine the convergence of the sequence $\left\{\frac{1}{3^n}\right\}$

a) Monotonic increasing
b) Oscillatory
c) Monotonic decreasing
d) None of these

(xiv) Choose the correct option. The sequence $\left\{\frac{3n+1}{n+2}\right\}$ is

a) Bounded
b) Unbounded
c) Divergent
d) None of these

(xv) Choose the correct value of $\lim_{(x,y)\to(0,0)} \frac{x^3-y^3}{x-y} =$

a) 0
b) 1
c) 1/2
d) None of these

Group-B

(Short Answer Type Questions) 3 x 5=15

2. Establish that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$. (3)

3. Construct the Maclaurin Series expansion of cos x. (3)

4. Examine the convergence of the series $\sum_{n=1}^{\infty} \frac{1}{n^2+5n}$ (3)

5. (3)

Without expanding illustrate that $\begin{vmatrix} 0 & b-a & c-a \\ a-b & o & c-b \\ a-c & b-c & 0 \end{vmatrix} = 0.$

6. Determine the matrix representation for the given linear transformation T relative to the ordered bases B and C, where T: V → V, where V = span $\{e^{2x}, e^{-3x}\}$, given by T(f) = f ′ with B= $\{e^{2x} - 3e^{-3x}, 2e^{-3x}\}$ and C= $\{e^{2x} + e^{-3x}, -e^{2x}\}$. (3)

OR

Determine whether the set of vectors $\{(a, b) \in \mathbb{R}^2 : b = a + 2\}$ is a vector space. (3)

Group-C

(Long Answer Type Questions) 5 x 6=30

7. A mapping $T: R^3 \to R^3$ is defined by $T(x_1, x_2, x_3) = (x_1 + x_2 + x_3, 2x_1 + x_2 + 2x_3, x_1 + 2x_2 + x_3), (x_1, x_2, x_3) \in R^3$. Justify that T is a linear mapping. Find $KerT$ and the dimension of the $KerT$. (5)

8. Calculate $\iint xy(x + y)\, dxdy$ over the area bounded by $y = x^2$ and $y = x$. (5)

9. If $\begin{vmatrix} x^3 + 3x & x-1 & x+3 \\ x+1 & 1-2x & x-4 \\ x-2 & x+4 & 3x \end{vmatrix} = ax^4 + bx^3 + cx^2 + dx + e$ be an identity in x where a, b, c, d, e are constants, then calculate the value of e. (5)

10. Illustrate $\int_0^\infty e^{-x^4} x^2 dx * \int_0^\infty e^{-x^4} dx = \frac{\pi}{8\sqrt{2}}$. (5)

11. Establish that $\lim_{n\to\infty} \left(\frac{1}{\sqrt{n^2+1}} + \frac{1}{\sqrt{n^2+2}} + \cdots + \frac{1}{\sqrt{n^2+n}}\right) = 1$ (5)

12. Evaluate the eigenvalues and eigenvectors of the matrix $\begin{bmatrix} 0 & 0 & -1 \\ 1 & 0 & 0 \\ 1 & 1 & -1 \end{bmatrix}$. (5)

OR

(5)

Deduce the set of vectors $v_1 = (1,3), v_2 = (3,1), v_3 = (1,2), v_4 = (1,1)$ to obtain a basis of $\mathbb{R}^4$.

****