BRAINWARE UNIVERSITY

Term End Examination 2023

Programme – B.Tech.(CSE)-2018/B.Tech.(ECE)-2018/B.Tech.(ECE)-2019/B.Tech.(CSE)-2019/B.Tech.(CSE)-2020/B.Tech.(ECE)-2020/B.Tech.(RA)-2022

Course Name – Calculus/Calculus & Linear Algebra

Course Code - BMAT010101/BSC(ECE)101/BSC(CSE)101/BSCR102

(Semester I)

Full Marks : 60 **Time : 2:30 Hours**

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group-A

(Multiple Choice Type Question) 1 x 15=15

1. *Choose the correct alternative from the following :*

(i) Write the value of $\Gamma\left(\frac{1}{3}\right)\Gamma\left(\frac{2}{3}\right)$ is

a) $\frac{2\pi}{\sqrt{3}}$ b) $\frac{3\pi}{\sqrt{2}}$

c) $\frac{\pi}{\sqrt{3}}$ d) $\frac{\pi}{\sqrt{2}}$

(ii) Examine, $B(1,1) =$

a) π b) 1

c) $\frac{\sqrt{\pi}}{2}$ d) None of these

(iii) If $u = \log\frac{x^2}{y}$ then calculate $xu_x + yu_y =$

a) U b) 2

c) 1 d) 2u

(iv) For $x > 0$, calculate $\lim_{x\to 0}\frac{\sin x}{\sqrt{x}} =$

a) 1 b) 2

c) 0 d) None of these

(v) Select the value of the triple integral $\int_0^1\int_5^8\int_0^9 dx\,dy\,dz$ is

a) 25
b) 27
c) 1
d) 3

(vi) Select the value of $\int_0^{\frac{\pi}{2}}\int_0^1 r^2 \sin\theta\, dr\, d\theta$ as

a) 0
b) 1/3
c) -1/3
d) none of these

(vii) If a function $f:[a,b]\to \square$ be continuous on $[a,b]$ and differentiable on (a,b). If $f'(x)=0$ for all $x\in(a,b)$ then tell the value of f is

a) 1
b) 0
c) Constant
d) None of these

(viii) Select the value of $\iint_R xy(x^2+y^2)\,dxdy$ over $R:[0,a;0,b]$

a) $ax^2/2 - x^5/30$
b) $ax^2/2 - x^3/6$
c) $ax^2/2$
d) [redacted] $\frac{1}{8}a^2b^2(a^2+b^2)$

(ix) Tell the Cauchy's form of remainder in Taylor's theorem is

a) [redacted] $\frac{h^n(1-\theta)^{(n-1)}}{(n-1)!}f^n(a+\theta h)$
b) $\frac{h^n(1-\theta)^{(n-p)}}{p(n-1)!}f^n(a+\theta h)$
c) $\frac{h^n}{n!}f^n(a+\theta h)$
d) None of these

(x) Select the Lagrange's form of remainder in Taylor's theorem is

a) $\frac{h^n(1-\theta)^{(n-1)}}{(n-1)!}f^n(a+\theta h)$
b) $\frac{h^n(1-\theta)^{(n-p)}}{p(n-1)!}f^n(a+\theta h)$
c) $\frac{h^n}{n!}f^n(a+\theta h)$
d) None of these

(xi) Select the value of $\int_C (xdx-dy)$, where C is the line joining (0,1) to (1,0) is

a) 3/2
b) 1/2
c) 0
d) 2/3

(xii) Select from the following: If $\phi(x,y)=x^2-2xy+y^3$, then $\vec{\nabla}\phi$ at $(2,3)$ is

a) $-2\hat{i}+2\hat{j}$
b) $-2\hat{i}-2\hat{j}$
c) $2\hat{i}-2\hat{j}$
d) $2\hat{i}+2\hat{j}$

(xiii) Test the improper integral

$$\int_1^\infty f(x)dx, \text{ where } f(x)=\begin{cases}\frac{1}{x^2}, & \text{if } x \text{ be rational} \geq 1\\ -\frac{1}{x^2}, & \text{if } x \text{ be irrational} > 1\end{cases} \text{ is}$$

a) Convergent
b) Divergent

c) <p style="text-align: left;"> 1

d) None of these

(xiv) Test the improper integral $\int_0^{\infty} \frac{\cos mx}{x^2+a^2} dx,\ m>0,\ a>0$ is

a) Convergent

b) Divergent

c) 1

d) None of these

(xv) Select the evolute of the parabola $y^2 = 4ax$ is

a) $ax + 4y = 1$

b) $27ay^2 = 4(x-2a)^3$

c) $27ay = 4(x-2a)$

d) None of these

Group-B

(Short Answer Type Questions) 3 x 5=15

2. Show that $\frac{(b-a)}{\sqrt{1-a^2}} < \sin^{-1} b - \sin^{-1} a < \frac{(b-a)}{\sqrt{1-b^2}}$ if $0 < a < b < 1$ (3)

3. State Maclaurin's theorem for expansion of a function in infinite series. Expand the functions e^{ax} in power of x in an infinite series. (3)

4. Apply Maclaurin's theorem to the function $f(x) = (1+x)^4$ to deduce that $(1+x)^4 = 1 + 4x + 6x^2 + 4x^3 + x^4$. (3)

5. Show that the series $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots$ is conditionally convergent. (3)

6. Prove that every convergent sequence is bounded. (3)

OR

If $u_n = \frac{1}{1.2} + \frac{2}{2.3} + \frac{1}{3.4} + \cdots + \frac{1}{n(n+1)}$, then show that the sequence $\{u_n\}$ is monotonically increasing and bounded. (3)

Group-C

(Long Answer Type Questions) 5 x 6=30

7. Examine the following function for extreme values and saddle points : (5)

$$f(x, y) = x^3 + 3xy^2 - 3y^2 - 3x^2 + 4$$

8. a. Evaluate $\iiint (x+y+z+1)^4 dx dy dz$ over the region bounded by (5)

$x \geq 0,\ y \geq 0,\ z \geq 0,\ x+y+z \leq 1$.

b. Show that $\int_0^{\pi/2} \sqrt{\tan\theta}\, d\theta = \frac{\pi}{\sqrt{2}}$.

9. Examine the convergence of the power series (5)

$$2x + \frac{3x^2}{8} + \frac{4x^3}{27} + \cdots \quad (x > 0)$$

10. If $u = xf\left(\frac{y}{x}\right) + g\left(\frac{y}{x}\right)$, then show that (5)

(i) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = xf\left(\frac{y}{x}\right)$

(ii) $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 0$.

11. Verify Greens's Theorem in the plane for $\oint_C [(y - \sin x)dx + \cos x dy]$ where C is the triangle whose vertices are (0, 0), $\left(\frac{\pi}{2}, 0\right)$ and $\left(\frac{\pi}{2}, 1\right)$. (5)

12. Show that the sequence $\left\{\left(1+\frac{1}{n}\right)^n\right\}$ is a monotone increasing sequence, bounded above. (5)

OR

Show that the series $\frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \ldots\ldots$ converges for $p > 1$ and diverges for $p \leq 1$. (5)

****