



BRAINWARE UNIVERSITY

Term End Examination 2023-2024 Programme - M.Sc.(MATH)-2022 Course Name – Functional Analysis Course Code - MSCMC301 (Semester III)

Brainware July Brain 25

Full Marks: 60

Time: 2:30 Hours

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group-A

(Multiple Choice Type Question)

1 x 15=15

- Choose the correct alternative from the following :
 - (i) Select the correct answer. Let $T: X \to Y$ be a bounded linear operator between normed spaces X and Y. Then T is
 - a) always continuous

- b) never continuous
- c) continuous at some points of X
- d) T can't be unbounded
- (ii) Select the correct answer. In an inner product space of dimension 2 if a set of two vectors are orthogonal to each other then the set
 - a) Linearly dependent

b) Linearly independent but not a basis

c) Basis

- d) None of these
- (iii) Select the correct answer.
 - a) Hilbert space is inner product space
- b) All inner product spaces are Hilbert spaces
- c) All Banach spaces are Hilbert spaces
- d) Other
- (iv) Select the correct answer. Let H be a separable Hilbert space then every orthonormal set in H is
 - a) Finite

b) countable

c) uncountable

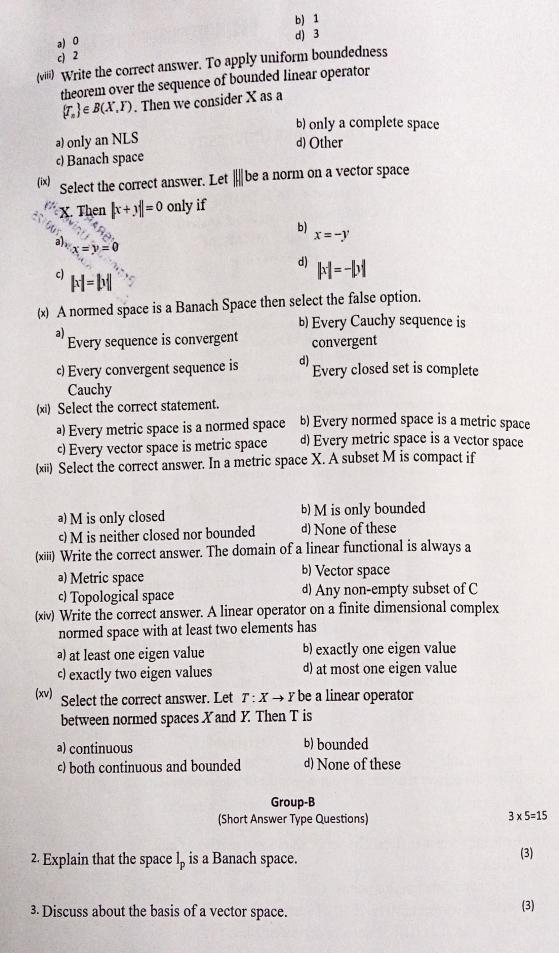
- d) other
- (v) Select the correct answer. The inner product of two vectors is a
 - a) real number

b) complex number

c) vector

- d) other
- (vi) Write the correct answer. The set of all rational number Q in R
 - a) dense

- b) nowhere dense
- c) have countable interior points
- d) other
- (vii) Select the correct answer. If set of vectors
 - $\{(1,0,0),(1,x,1),(x,0,1)\}$ is linearly dependent, then x is



5. Analyze that the space C[a, b] is not a Hilbert space. 6. Justify that not every norm linear space is a Banach space. (3) Let T be a linear operator. Then the null space is a vector space. Justify your (3) answer. Group-C 5 x 6=30 (Long Answer Type Questions) 7. Show that the set $S = \{(0,1,1),(1,0,1).(1,1,1)\}$ is a basis for \mathbb{R}^3 (5) 8. Examine the following statements. (5) For every linear operator $T: X \to Y$ we have (i)T(0)=0(ii)T(-x) = -Tx(iii)T(x-y) = Tx - Ty9. Let $T: X \to Y$ be a linear operator on X. Analyze that if the (5) image T(E) of every bounded subset $E \subset X$ is bounded in Y, then T is continuous at some point on X. 10. Explain the following statement. (5) For every norm-linear space X, addition is continuous in X, i.e., if $x_n \to x$ in X and $y_n \to y$, then $x_n + y_n \to x + y$. 11. Write and justify the polarization identity for inner product space. (5) (5) Let M be a complement subspace Y and $x \in X$ fixed. Then justify that z = x - y is orthogonal to Y.

(3)

(3)

4. Define Hilbert Space.

Justify that a linear subspace M of a Hilbert space H is close in H if and only if $M = M^{\perp \perp}$.	ed (5)