BRAINWARE UNIVERSITY

Term End Examination 2023-2024
Programme – M.Sc.(MATH)-2022
Course Name – Functional Analysis
Course Code - MSCMC301
(Semester III)

Full Marks : 60 **Time : 2:30 Hours**

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group-A

(Multiple Choice Type Question) 1 x 15=15

1. *Choose the correct alternative from the following :*

(i) Select the correct answer. Let $T : X \rightarrow Y$ be a bounded linear operator between normed spaces X and Y. Then T is

a) always continuous
b) never continuous
c) continuous at some points of X
d) T can't be unbounded

(ii) Select the correct answer. In an inner product space of dimension 2 if a set of two vectors are orthogonal to each other then the set

a) Linearly dependent
b) Linearly independent but not a basis
c) Basis
d) None of these

(iii) Select the correct answer.

a) Hilbert space is inner product space
b) All inner product spaces are Hilbert spaces
c) All Banach spaces are Hilbert spaces
d) Other

(iv) Select the correct answer. Let H be a separable Hilbert space then every orthonormal set in H is

a) Finite
b) countable
c) uncountable
d) other

(v) Select the correct answer. The inner product of two vectors is a

a) real number
b) complex number
c) vector
d) other

(vi) Write the correct answer. The set of all rational number Q in R

a) dense
b) nowhere dense
c) have countable interior points
d) other

(vii) Select the correct answer. If set of vectors $\{(1,0,0),(1,x,1),(x,0,1)\}$ is linearly dependent, then x is

a) 0
b) 1
c) 2
d) 3

(viii) Write the correct answer. To apply uniform boundedness theorem over the sequence of bounded linear operator $\{T_n\} \in B(X, Y)$. Then we consider X as a

a) only an NLS
b) only a complete space
c) Banach space
d) Other

(ix) Select the correct answer. Let $\|\cdot\|$ be a norm on a vector space X. Then $\|x+y\| = 0$ only if

a) $x = y = 0$
b) $x = -y$
c) $\|x\| = \|y\|$
d) $\|x\| = -\|y\|$

(x) A normed space is a Banach Space then select the false option.

a) Every sequence is convergent
b) Every Cauchy sequence is convergent
c) Every convergent sequence is Cauchy
d) Every closed set is complete

(xi) Select the correct statement.

a) Every metric space is a normed space
b) Every normed space is a metric space
c) Every vector space is metric space
d) Every metric space is a vector space

(xii) Select the correct answer. In a metric space X. A subset M is compact if

a) M is only closed
b) M is only bounded
c) M is neither closed nor bounded
d) None of these

(xiii) Write the correct answer. The domain of a linear functional is always a

a) Metric space
b) Vector space
c) Topological space
d) Any non-empty subset of C

(xiv) Write the correct answer. A linear operator on a finite dimensional complex normed space with at least two elements has

a) at least one eigen value
b) exactly one eigen value
c) exactly two eigen values
d) at most one eigen value

(xv) Select the correct answer. Let $T: X \to Y$ be a linear operator between normed spaces X and Y. Then T is

a) continuous
b) bounded
c) both continuous and bounded
d) None of these

Group-B

(Short Answer Type Questions) 3 x 5=15

2. Explain that the space l_p is a Banach space. (3)

3. Discuss about the basis of a vector space. (3)

4. Define Hilbert Space. (3)

5. Analyze that the space C[a, b] is not a Hilbert space. (3)

6. Justify that not every norm linear space is a Banach space. (3)

OR

Let T be a linear operator. Then the null space is a vector space. Justify your answer. (3)

Group-C

(Long Answer Type Questions) 5 x 6=30

7. Show that the set S={(0,1,1),(1,0,1).(1,1,1)} is a basis for R^3 (5)

8. Examine the following statements. (5)
For every linear operator $T: X \rightarrow Y$ we have
$(i) T(0) = 0$
$(ii) T(-x) = -Tx$
$(iii) T(x - y) = Tx - Ty$

9. Let $T: X \rightarrow Y$ be a linear operator on X. Analyze that if the image T(E) of every bounded subset $E \subset X$ is bounded in Y, then T is continuous at some point on X. (5)

10. Explain the following statement. (5)
For every norm-linear space X, addition is continuous in X, i.e., if $x_n \rightarrow x$ in X and $y_n \rightarrow y$, then $x_n + y_n \rightarrow x + y$.

11. Write and justify the polarization identity for inner product space. (5)

12. Let M be a complement subspace Y and $x \in X$ fixed. Then justify that $z = x - y$ is orthogonal to Y. (5)

OR

(5)

Justify that a linear subspace M of a Hilbert space H is closed in H if and only if $M = M^{\perp\perp}$.

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