BRAINWARE UNIVERSITY

Term End Examination 2023-2024
Programme – M.Sc.(MATH)-2022
Course Name – Differential Geometry
Course Code - MSCMC302
(Semester III)

Full Marks : 60 **Time : 2:30 Hours**

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group-A

(Multiple Choice Type Question) 1 x 15=15

1. *Choose the correct alternative from the following :*

(i) ds^2 for the surface $x^1 = u^1,\ x^2 = u^2,\ x^3 = f(u^1, u^2)$ is illustrate as

a) $ds^2 = (1+f_1^2)(du^1)^2 + 2f_1f_2du^1du^2 + (1+f_2^2)(du^2)^2$

b) $ds^2 = (2+f_1^2)(du^1)^2 + 2f_2du^1du^2 + (1+f_2^2)(du^2)^2$

c) $ds^2 = (2+f_1^2)(du^1)^2 + 2f_2du^1du^2$

d) None of these

(ii) Select the correct option. The shape of a surface with negative Gaussian curvature is:

a) Spherical
b) Cylindrical
c) Hyperbolic
d) Parabolic

(iii) ds^2 for the surface $x^1 = u^1\cos u^2,\ x^2 = u^1\sin u^2,\ x^3 = 0$ is illustrate as

a) $ds^2 = u^2(du^1)^2 + (u^1)^2(du^2)^2$

b) $ds^2 = (du^1)^2 + (u^1)^2(du^2)^2$

c) $ds^2 = (u^1)^2(du^2)^2$

d) None of these

(iv) If a curve lies on a plane, choose the nature of the curve

a) Skew curve
b) Plane curve
c) Straight line
d) None of these

(v) If $g_{ij} = 0$ for $i \neq j$, then $[i\,j, i] =$ _______________ (Identify the correct option)

a) $\frac{1}{2}\cdot\frac{\partial g_{ii}}{\partial x^{j}}$

b) $-\frac{1}{2g_{ii}}\cdot\frac{\partial g_{jj}}{\partial x^{i}}$

c) $-\frac{1}{2}\cdot\frac{\partial g_{jj}}{\partial x^{i}}$

d) $\frac{1}{2}\cdot\frac{\partial g_{ii}}{\partial x^{j}}$

(vi) For the right helicoid $r=(u\cos v, u\sin v, cv)$, the ds is illustrate as

a) $ds^2=(du)^2+(u^2+1)(dv)^2$

b) $ds^2=(du)^2+(u^2+v^2)(dv)^2$

c) $ds^2=(du)^2+(dv)^2$

d) None of these

(vii) If θ be the angle between two parametric curves, then evaluate $\cos\theta=$

a) $\frac{a_{12}}{\sqrt{a_{11}a_{22}}}$

b) $\frac{a_{11}}{\sqrt{a_{11}a_{22}}}$

c) $\frac{a_{22}}{\sqrt{a_{11}a_{22}}}$

d) None of these

(viii) If A_m is a covariant vector , then determine $\frac{\partial A_m}{\partial x^s}-\frac{\partial A_s}{\partial x^m}$

a) Symmetric

b) Skew-symmetric

c) Constant

d) None of these

(ix) If θ be the angle between two parametric curves, then evaluate $\sin\theta=$

a) $\frac{a_{22}}{\sqrt{a_{11}a_{22}}}$

b) $\frac{\sqrt{a}}{\sqrt{a_{11}a_{22}}}$

c) $\frac{\sqrt{a_{11}}}{\sqrt{a_{11}a_{22}}}$

d) None of these

(x) If A_m is a covariant vector , determine $\frac{\partial A_m}{\partial x^s}-\frac{\partial A_s}{\partial x^m}$

a) A tensor of type (0,2)

b) A tensor of type (1,3)

c) A tensor of type(0,1)

d) A tensor of type (1,1)

(xi) If A_{ij} is a skew-symmetric tensor, identify $\left(\delta^i_{\ j}\delta^k_{\ i}-\delta^i_{\ i}\delta^k_{\ j}\right)A_{ik}=$

a) 1

b) 0

c) n

d) None of these

(xii) If A_{mn} is a skew-symmetric tensor and B^i is a contravariant vector, identify $A_{mn}B^mB^n=$

a) 0

b) 1

c) n

d) m

(xiii) Select the correct option. A geodesic curve__________

a) Is a locally length minimizing curve

b) Is not an auto parallel curve

c) Is a locally length maximizing curve

d) None of these

(xiv) If $\lambda^\alpha, \mu^\alpha$ are two unit vectors such that the rotation $\lambda^\alpha, \mu^\alpha$ is positive, then select the correct option

a) $\sin\theta=\in_{\alpha\beta}\lambda^\alpha\mu^\beta$

b) $\cos\theta=\in_{\alpha\beta}\lambda^\alpha\mu^\beta$

c) $\tan\theta = \epsilon_{\alpha\beta}\lambda^{\alpha}\mu^{\beta}$

d) None of these

(xv) If a helix can always be drawn on the surface of a right circular cylinder, then the helix is named as

a) Double helix

b) Circular helix

c) Helix

d) None of these

Group-B

(Short Answer Type Questions) 3 x 5=15

2. Define tangent vector of a curve. (3)

3. Evaluate $\delta_j^i \delta_i^k A^{jl}$. (Range of indices 1 to n) (3)

4. Let $I_1 = \left(0, \frac{\pi}{2}\right)$ and $I_2 = (0,1)$. Let $R_1(u) = (2\cos^2 u, \sin 2u, 2\sin u)$ be defined on I_1. If $\phi: I_2 \to I_1$ is defined as $u = \phi(v) = \sin^{-1}(v)$, write the parametric representation $R_2(v)$ equivalent to $R_1(u)$. (3)

5. Show that the following are the two equivalent representations of a circular helix. (3)

$$(i)\ R_1(u) = (a\cos u,\ a\sin u, bu), \qquad u \in I_1 = [0,\pi)$$

$$(ii)\ R_2(v) = \left(a\frac{1-v^2}{1+v^2}, \frac{2av}{1+v^2}, 2b\tan^{-1} v\right), v \in I_2 = [0,\infty).$$

6. Justify that a curve is a plane curve if $[\dot{r}, \ddot{r}, \dddot{r}] = 0$. (3)

OR

Evaluate δ_i^i. (Range of indices 1 to n) (3)

Group-C

(Long Answer Type Questions) 5 x 6=30

7. Define principal normal and binormal. (5)

8. Describe what do you mean by self-intersection point of a curve. Describe what do you mean by simple closed curve in $\mathbb{R}^2$. (5)

9. Evaluate the torsion of $r = (a\cos\theta,\ a\sin\theta, a\theta\cot\alpha)$. (5)

10. Evaluate the torsion of the curves given by $r = \{a(3u - u^3), 3au^2, a(3u + u^3)\}$. (5)

11. Evaluate the curvature of $r = (a\cos\theta,\ a\sin\theta, a\theta\cot\alpha)$. (5)

12. Write Wirtinger's inequality and write the definition of surface patch. (5)

OR

(5)

If $|a_j^i| = \begin{vmatrix} a_1^1 & a_2^1 & a_3^1 \\ a_1^2 & a_2^2 & a_3^2 \\ a_1^3 & a_2^3 & a_3^3 \end{vmatrix}$ and $|b_j^i| = \begin{vmatrix} b_1^1 & b_2^1 & b_3^1 \\ b_1^2 & b_2^2 & b_3^2 \\ b_1^3 & b_2^3 & b_3^3 \end{vmatrix}$, then justify that $|a_j^i||b_m^k| = |c_j^i|$, where $c_m^i = a_p^i a_m^p$.

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