BRAINWARE UNIVERSITY

Term End Examination 2023-2024
Programme – M.Sc.(MATH)-2022
Course Name – Integral Equations & Calculus of Variations
Course Code - MSCMC303
(Semester III)

Full Marks : 60 Time : 2:30 Hours

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group-A
(Multiple Choice Type Question) 1 x 15=15

1. *Choose the correct alternative from the following :*

(i) The homogeneous integral equation $y(x) = \lambda \int_0^1 (3x-2)\, y(t)dt$ can be classified as

a) zero characteristic numbers
b) four characteristic numbers
c) two characteristic numbers
d) None of these

(ii) Illustrate the Laplace transformation of $t^{5/2}$

a) $\frac{15}{8}\frac{\sqrt{\pi}}{s^{7/2}}$
b) $\frac{15}{8}\frac{\sqrt{\pi}}{s^{5/2}}$
c) $\frac{7}{8}\frac{\sqrt{\pi}}{s^{7/2}}$
d) $\frac{15}{8}\frac{\sqrt{\pi}}{s^{3/2}}$

(iii) Illustrate the solution of the integral equation $u(x) = x \int_0^{1/2} u(t)dt$ (Fredholm Integral equation of 2nd kind) is

a) $u(x) = x + \frac{1}{2}$
b) $u(x) = x + \frac{1}{4}$
c) $u(x) = x - \frac{1}{4}$
d) $u(x) = x - \frac{1}{2}$

(iv) Predict the obvious solution of homogeneous Fredholm integral equation $y(x) = \lambda \int_a^b K(x,t)\, y(t)dt$.

a) $y(x) = 0$
b) $y(x) = 1$
c) $y(x) = 2$
d) None of these

(v) The integral equation obtained from $y'' - xy' + y = 0$ with $y(0) = 1, y'(0) = -1$ can be expressed as

a) Volterra equation of 1st kind
b) Fredholm equation of 1st kind
c) Volterra equation of 2nd kind
d) Fredholm equation of 2nd kind

(vi) Determine the value of α for which the integral equation $u(x)=\alpha\int_0^1 e^{x-t}u(t)\,dt$, has a non–trivial solution is

a) -2
b) -1
c) 1
d) 2

(vii) Identify the definition of Resolvent Kernel?

a) $y(x) = f(x) + \lambda\int_a^b R(x,t;\lambda)f(t)dt$
b) $y(x) = f(x) + \lambda\int_a^x \Gamma(x,t;\lambda)\,f(t)dt$
c) Both (a) & (b)
d) Neither (a) nor (b)

(viii) Calculate the function that is a solution of the Volterra type equation

$$f(x)=x+\int_0^x \sin(x-t)\,f(t)\,dt$$

a) $x+\frac{x^3}{3}$
b) $x-\frac{x^3}{3}$
c) $x+\frac{x^3}{6}$
d) $x-\frac{x^3}{6}$

(ix) Select the correct option. The resolvent kernel of the integral equation

$$y(x)=x-\int_0^x xt^2 y(t)dt, x>0$$

is

a) $xt^2e^{-\frac{x^4-t^4}{4}}$
b) $xt^2e^{-\frac{x^2-t^2}{2}}$
c) $xte^{-\frac{x^4-t^4}{4}}$
d) None of these

(x) Select the correct option. An integral equation is an equation in which the unknown function appears under one or more integral signs.

a) always true
b) always false
c) maybe true
d) None of these

(xi) Choose the correct option. Geodesics on a sphere of radius a are its

a) great circles
b) circles
c) parabola
d) None of these.

(xii) Select the correct option. The general form of a linear integral equation is

a) $y(x) = \int_a^b K(x,t)\,[y(t)]^2dt$
b) $g(x)y(x) = f(x) + \lambda\int_{a(x)}^{b(x)} K(x,t)y(t)dt$
c) $\int_a^b K(x,t)\,y(t)dt = f(x)$
d) None of these

(xiii) Solve the integral equation

$$y(x)=\cos x+\lambda\int_0^{\pi} \sin x\, y(t)\,dt$$

a) $y(x)=\cos x$, provided $\lambda\neq\frac{1}{2}$
b) $y(x)=\sin x$, provided $\lambda\neq\frac{1}{2}$
c) $y(x)=\tan x$, provided $\lambda\neq\frac{1}{2}$
d) None of these

(xiv) Select the correct option. If a functional I[y(x)] having a variation attains a ....on $y=y_0(x)$ then at $y=y_0(x)$, $\delta(I)=0$

a) maximum
b) minimum
c) maximum or minimum
d) none of these

(xv) If g(x)=0, in $g(x)y(x) = f(x) + \lambda \int_a K(x,t)\, y(t)dt$, then the equation can be identified as

a) linear integral equation of 1st kind
b) linear integral equation of 2nd kind
c) linear integral equation of 3rd kind
d) None of these

Group-B
(Short Answer Type Questions) 3 x 5=15

2. Define Volterra integral equation with an Example. (3)

3. Show that the given functions is the solutions of the corresponding integral equation (3)
$y(x) = 1 - x\,;\ \int_0^x e^{x-t}\, y(t)dt = x$

4. Define Laplace transformation and using the definition illustrate the Laplace transformation of $F(t) = 1$ (3)

5. Illustrate the solution of the integral equation (3)

$$\int_0^x f(t)\, f(x-t)\, dt = 16\sin 4x$$

6. Explain the Euler's equation of a functional with an example. (3)

OR

Deduce the Euler's equation for the extremals of the functional (3)

$$\int_{x_1}^{x_2} \{5y^3 - 15y' + 2y'^2\}\, dx$$

Group-C
(Long Answer Type Questions) 5 x 6=30

7. Express the differential equation with the given initial conditions: $y''' - 2xy = 0;\ y(0) = \frac{1}{2}, y'(0) = y''(0) = 1$ into an integral equation. (5)

8. Solve the following integral equation (5)

$$y(x) = \cos x + \lambda \int_0^{\pi} \sin x\, y(t)dt$$

9. Calculate the eigen values and eigen functions of the homogeneous integral equation (5)

$$y(x) = \lambda \int_0^1 e^x e^t y(t)dt$$

10. If $\alpha(x)$ is continuous in $[a,b]$ and if $\int_a^b \alpha(x)h(x)dx = 0$ for every function $h(x) \in C[a,b]$ such that $h(a) = 0 = h(b)$, then deduce that $\alpha(x) = 0$ for all $x \in [a,b]$. (5)

11. Calculate the general solution of the Euler's equation for the functional (5)

$$\int_a^b \frac{1}{y} \sqrt{\left\{1 + \left(\frac{dy}{dx}\right)^2\right\}}\, dx$$

12. Evaluate the extremes of the functional (5)

$\int_0^{\frac{\pi}{4}} (y''^2 - y^2 + x^2)\, dx : y(0) = 0, y'(0) = 1, y\left(\frac{\pi}{4}\right) = y'\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}.$

OR

Choose the curve on which functional $\int_0^1 [(y')^2 + 2y]dx$ with boundary conditions $y(0) = 1$ and $y(1) = 0$ can be extremized, if exists. (5)

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