BRAINWARE UNIVERSITY

Term End Examination 2023-2024
Programme – M.Sc.(MATH)-2022
Course Name – Analytical Number Theory
Course Code - MSCME301C
(Semester III)

Full Marks : 60

Time : 2:30 Hours

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group-A
(Multiple Choice Type Question)   1 x 15=15

1.   Choose the correct alternative from the following :

(i)   Select the correct option. The Kronecker's Lemma is primarily concerned with which type of numbers?

a) Prime numbers

b) Imaginary numbers

c) Rational numbers

d) Irrational numbers

(ii) Determine the correct statement about the upper bound of the Riemann Zeta function?

a) The Riemann Zeta function is bounded by a constant value.

b) The Riemann Zeta function is unbounded and grows infinitely as the input increases.

c) The upper bound of the Riemann Zeta function is unknown.

d) The Riemann Zeta function is bounded by a logarithmic function.

(iii) Select the correct option: The Dirichlet series associated with the Riemann zeta function is convergent for:

a) Re(s) > 0

b) Re(s) > -1

c) Re(s) > 1

d) Re(s) > -2

(iv) Determine the lower bound for the Riemann Zeta function?

a) The Riemann Zeta function is always positive, so it has no lower bound

b) The Riemann Zeta function is bounded below by a constant value.

c) The lower bound of the Riemann Zeta function is unknown.

d) The Riemann Zeta function is bounded below by a logarithmic function.

(v)   Choose the correct statement.

a) Dirichlet series are unrelated to arithmetic functions.

b) Every arithmetic function has a corresponding Dirichlet series.

c) Dirichlet series can be used to define arithmetic functions.

d) Arithmetic functions cannot be expressed as Dirichlet series.

(vi) Conclude the statement that is about Euler products

a) Euler products are only applicable to arithmetic functions.

b) Euler products are derived from infinite geometric series.

c) Euler products provide an alternative representation of certain Dirichlet series.

d) Euler products are limited to functions with real inputs.

(vii) Identify the correct option: In Mellin transformation Dirichlet series should be

a) Divergent

b) Convergent

c) Oscillatory

d) None of these

(viii) Conclude the true statement for the divisor function $\sigma(n)$ and the sum of divisors function $d(n)$

a) $\sigma(n) = d(n) - n$

b) $\sigma(n) = d(n) + n$

c) $\sigma(n) = 2 * d(n)$

d) $\sigma(n) = d(n) * n$

(ix) Apart from the Mellin transformation, identify the option which concludes the convergence of the Dirichlet series

a) Wintner's theorem

b) Kronecker's lemma

c) Stirling's formula

d) Moebius Function

(x) Convolution method can be identified as

a) Approximating an arithmetic function by a simpler arithmetic function

b) Approximating the partial sum by a simpler arithmetic function

c) Evaluating an arithmetic function by partial sum

d) None of these

(xi) The Euler summation formula can be used to evaluate the sum of the series

a) The harmonic series $(1 + 1/2 + 1/3 + 1/4 + ...)$.

b) The alternating series $(1 - 1/2 + 1/3 - 1/4 + ...)$.

c) The geometric series $(1 + 2 + 4 + 8 + ...)$.

d) The factorial series $(1! + 2! + 3! + 4! + ...)$.

(xii) Identify the correct option. A divisor of an integer means

a) A positive divisor

b) A negative divisor

c) A zero divisor

d) None of these

(xiii) A negative integer can be defined as

a) a prime

b) not a prime

c) a composite number

d) coprime

(xiv) Every integer >1 can be recognized as

a) Product of primes

b) Addition of primes

c) Division of primes

d) None of these

(xv) The factorization of a prime number can be defined by

a) Fundamental Theorem of Arithmetic

b) Fundamental Theorem of Algebra

c) Division theorem

d) None of these

Group-B
(Short Answer Type Questions)

3 x 5=15

2. Show that the nth prime pn satisfies pn ~ n log n as n → ∞. (3)
3. Define prime number with example (3)
4. Evaluate the absolutely convergence of Dirichlet series. (3)
5. Explain disk of convergence and radius of convergence. (3)
6. Predict an application of partial sums of the Euler phi function (3)

OR

Evaluate the partial sum of Euler phi function with proper significance of each symbol. (3)

Group-C
(Long Answer Type Questions)

5 x 6=30

7. Compute Chebyshev estimate.

(5)

8. Explain the summation by parts formula. (5)
9. Evaluate Wintner's mean value theorem. (5)
10. Evaluate an application of Mellin transform representation of Dirichlet series. (5)
11. Conclude the Carmichael's conjecture. (5)
12. Evaluate Moebius identity. (5)

OR

Justify Stirling's formula. (5)

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