



BRAINWARE UNIVERSITY

Term End Examination 2023-2024
Programme – M.Sc.(MATH)-2022
Course Name – Dynamical systems
Course Code - MSCME302B
(Semester III)

Library
Brainware University
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Kolkata, West Bengal-700125

Full Marks : 60

Time : 2:30 Hours

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group-A

(Multiple Choice Type Question)

1 x 15=15

1. Choose the correct alternative from the following :

(i) Choose the correct option. In a Lyapunov function $V(x)$, a point x^* is considered stable if $V(x^*)$ is:

- a) Positive definite
- b) Negative definite
- c) Non-positive
- d) Non-negative

(ii) Select the correct option, if $A = \begin{pmatrix} a & b \\ 0 & a \end{pmatrix}$ then e^A is equal to

- a) $e^a \begin{pmatrix} a & b \\ b & a \end{pmatrix}$
- b) $e^a \begin{pmatrix} 1 & b \\ b & 1 \end{pmatrix}$
- c) $e^a \begin{pmatrix} 1 & -b \\ b & 1 \end{pmatrix}$
- d) $e^a \begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix}$

(iii) If a discrete dynamical system has two fixed points, one attracting and one repelling, recognize the best describes the behaviour of trajectories near these fixed points.

- a) Trajectories are attracted to the attracting fixed point and repelled from the repelling fixed point
- b) Trajectories are repelled from both fixed points
- c) Trajectories are attracted to both fixed points
- d) Trajectories do not converge near any fixed point

(iv) Select the correct option; a function $f(x)$ satisfies the Lipschitz condition on an interval $[a, b]$ if there exists a constant K such that:

- a) $|f(x)| \leq K$ for all $x \in [a, b]$
- b) $|f'(x)| \leq K$ for all $x \in [a, b]$
- c) $|f(x) - f(y)| \leq K|x - y|$ for all $x, y \in [a, b]$
- d) $|f''(x)| \leq K$ for all $x \in [a, b]$

(v) Select the primary characteristic behavior exhibited by the Lienard system.

- a) Stable equilibrium points
- b) Unbounded growth
- c) Exponential growth of trajectories
- d) Chaotic behavior

(vi) Select the critical points of the nonlinear systems $\frac{dx}{dt} = x - xy$, $\frac{dy}{dt} = y - x^2$

- a) $(0,0), (\pm 1,1)$
b) $(0,0), (1, \pm 2)$
c) $(0,0), (-1, \pm 1)$
d) None of these

(vii) Choose the number of linearly independent eigenvectors a matrix can have if it has n distinct eigenvalues:

- a) n
b) $n - 1$
c) It depends on the size of the matrix
d) It depends on the values of the eigenvalues

(viii) Choose the matrix that will always have at least one eigenvalue equal to zero:

- a) Identity matrix
b) Zero matrix
c) Diagonal matrix with all non-zero entries
d) Skew-symmetric matrix

(ix) Write the correct statements for a positive definite matrix

- a) All of its eigenvalues are positive
b) It does not have any eigenvalues
c) All of its eigenvalues are negative
d) It has only one eigenvalue

(x) If a matrix A has eigenvalues $\{1, 2, 3\}$, Select the eigenvalues of A^2

- a) $\{1, 2, 3\}$
b) $\{1, 4, 9\}$
c) $\{2, 4, 6\}$
d) $\{1, 8, 27\}$

(xi) If a matrix A is invertible, select the correct statement about its eigenvalues

- a) They are all zero
b) They are all positive
c) None of them are zero
d) They are all equal to 1

(xii) If a matrix has an eigenvalue of multiplicity 3, Select the maximum number of linearly independent eigenvectors it can have?

- a) 0
b) 1
c) 2
d) 3

(xiii) Recognize the characteristic of chaotic behaviour in a discrete dynamical system:

- a) Sensitivity to initial conditions
b) Stable fixed points
c) Converging orbits
d) Periodic oscillations

(xiv) Select the correct option for the fixed points of a map in a discrete dynamical system:

- a) The map is undefined
b) The map is constant
c) The map remains unchanged after iteration
d) The map diverges to infinity

(xv) Select the equation that represents the fixed point of a discrete dynamical system:

- a) $f(x) = x$
b) $f'(x) = x$
c) $f(x) = 0$
d) $f'(x) = 0$

Group-B

(Short Answer Type Questions)

3 x 5 = 15

2. Write the definition of asymptotically stability.

(3)

3. Compute the value of k for which the following matrix is positive definite

$$\begin{pmatrix} 2 & -4 \\ -4 & k \end{pmatrix}$$

(3)

4. Define orbit of period k .

(3)

5. Show that $(0,0)$ is an unstable critical point of (3)

$$\begin{aligned}\frac{dx}{dt} &= -y + x^3 \\ \frac{dy}{dt} &= x + y^3\end{aligned}$$

6. Calculate the characteristic polynomial of $A = \begin{pmatrix} 5 & -2 \\ 4 & -4 \end{pmatrix}$ (3)

OR

Evaluate all the fixed points of the

$$\begin{aligned}T(x) &= 2x, 0 \leq x \leq \frac{1}{2} \\ &= 2(1-x), \frac{1}{2} \leq x \leq 1\end{aligned}$$

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Group-C

(Long Answer Type Questions)

5 x 6=30

7. Defining tent map, identify all fixed points of tent map. (5)

8. Justify whether each of the following functions is positive definite, negative definite, or neither (5)

$$x^2 - xy - y^2$$

9. Discuss if the linear system $\dot{x} = Ax$ has a saddle, node, focus or center at the origin and determine the stability of each node or focus: $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ (5)

10. Analyze the stable, unstable and center subspaces of the linear system (5)

$$\dot{x} = A \begin{pmatrix} 2 & 3 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} x$$

11. Classify the equilibrium points (as sinks, sources or saddles) of the nonlinear system $\dot{x} = f(x)$ with $f(x)$ given by (5)

$$\begin{pmatrix} x_2 - x_1 \\ kx_1 - x_2 - x_1x_3 \\ x_1x_2 - x_3 \end{pmatrix}$$

12. Evaluate the derivative of the functions $f(x) = \begin{pmatrix} x_1 + x_1x_2^2 + x_1x_3^2 \\ -x_1 + x_2 - x_2x_3 + x_1x_2x_3 \\ x_2 + x_3 - x_1 \end{pmatrix}$ (5)

OR

Consider

$$\begin{aligned} \dot{x} &= x - y - x(x^2 + 5y^2) \\ \dot{y} &= x + y - y(x^2 + y^2) \end{aligned}$$

evaluate the fixed point at the origin. (5)

COMPLETE