BRAINWARE UNIVERSITY

Term End Examination 2023
Programme – M.Sc.(MATH)-2021
Course Name – Applied Numerical Analysis
Course Code - MSCMC402
(Semester IV)

Full Marks : 60 **Time : 2:30 Hours**

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group-A

(Multiple Choice Type Question) 1 x 15=15

1. *Choose the correct alternative from the following :*

(i) Choose the correct option. The Jacobi's method is a method of solving a matrix equation on a matrix that has no zeroes along

a) Leading diagonal
b) Last column
c) Last row
d) Non-leading diagonal

(ii) Choose the correct option. The power method is applicable if the matrix has

a) n dependent eigenvectors
b) n independent eigenvectors
c) (n-1) independent eigenvectors
d) None of these

(iii) Select the correct assumption of Jacobi's method?

a) The coefficient matrix has no zeros on its main diagonal
b) The rate of convergence is quite slow compared with other methods
c) Iteration involved in Jacobi's method converges
d) The coefficient matrix has zeroes on its main diagonal

(iv) Choose the correct option. Which of these is correct for the multipoint method?

a) multiple derivatives at each time step
b) only one evaluation of derivative per time step
c) order of accuracy is restricted to four
d) extremely unstable

(v) <p style="text-align: left;"> How many assumptions are there in Jacobi's mathod. Judge the correct option.

a) 2
b) 3
c) 4
d) 5

(vi) Choose the correct option. The first two steps of the fourth-order Runge-Kutta method use _________

a) Euler methods
b) Forward Euler method
c) Backward Euler method
d) Explicit Euler method

(vii) What is Eigen value? Select the correct option

a) A vector obtained from the coordinates

b) A matrix determined from the algebraic equations

c) A scalar associated with a given linear transformation

d) It is the inverse of the transform

(viii) Choose the correct option. For $\frac{dy}{dx} = x + y$, and $y(0) = 1$, the value of $y(1.1)$ according to the Euler method is [taking $h = 0.1$]

a) 0.1

b) 0.3

c) 1.1

d) 0.9

(ix) Separate the correct answer. To find the values at the current time-step, the Crank-Nicolson scheme uses ___________

a) t-Δ t and t-2Δ t steps

b) t-Δt and t+Δ t steps

c) t+Δ t and t+2Δ t steps

d) t and t+Δt steps

(x) Select the correct option. Which of the following is another name for Jacobi's method?

a) Displacement method

b) Simultaneous displacement method

c) Simultaneous method

d) Diagonal method

(xi) Classify the other name for factorization method?

a) Doolittle's Method

b) Lin Bairstow Method

c) Muller's Method

d) Decomposition Method

(xii) Select the correct option. Robin boundary condition is also known as

a) first-type boundary condition

b) second type boundary condition

c) zero type boundary condition

d) third type boundary condition

(xiii) Select the correct option. When solving a 1-Dimensional wave equation using variable separable method, we get the solution if

a) k is positive

b) k is negative

c) k is 0

d) k can be anything

(xiv) Select the correct boundary condition which specifies the value of the normal derivative of the function

a) Neumann boundary condition

b) Neument boundary condition

c) Neumornn boundary condition

d) Deumann boundary condition

(xv) Select the correct option. For the transient convection problems, the Crank-Nicolson scheme is stable when

a) CFLconv≤2

b) $CFL^{conv}\leq 1$

c) $CFL^{conv}\geq -2$

d) $CFL^{conv}\geq -1$

Group-B

(Short Answer Type Questions) 3 x 5=15

2. Differentiate the need of using numerical methods as compared to analytical methods for solving a differential equation. (3)

3. Describe Jacobi's method to find eigenvalues and eigenvectors of a symmetric matrix. (3)

4. Apply the Power method to find the largest eigenvalue and eigenvector of the given matrix (3)

$\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ upto 2 decimal places.

5. Analyze the stability of the parabolic equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ which is approximated by finite difference scheme (3)

$$\frac{1}{k}\left(u_{p,q+1} - u_{p,q}\right) = \frac{1}{h^2}\left(u_{p-1,q+1} - 2u_{p,q+1} + u_{p+1,q+1}\right) \text{ at } (ph, qk).$$

6. Conclude that the region in which the following equation $x^3 \frac{\partial^2 u}{\partial x^2} + 27\frac{\partial^2 u}{\partial y^2} + 3\frac{\partial^2 u}{\partial x \partial y} + 5u = 0$ acts as an elliptic equation is $x > \left(\frac{1}{12}\right)^{\frac{1}{3}}$. (3)

OR

Analyze the stability of the parabolic equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ which is approximated by finite difference scheme (3)

$$\frac{1}{k}\left(u_{p,q+1} - u_{p,q}\right) = \frac{1}{h^2}\left(u_{p-1,q+1} - 2u_{p,q+1} + u_{p+1,q+1}\right) \text{ at } (ph, qk).$$

Group-C

(Long Answer Type Questions) 5 x 6=30

7. Show that the Crank-Nicholson implicit scheme is unconditionally stable. (5)

8. Analyze the Du Fort and Frankel method. (5)

9. Analyze the two iterative processes (i) Jacobi's method (ii) Gauss-seidal's method and compare these methods. (5)

10. Illustrate the boundary value problem ut = uxx under the conditions $u(0, t) = u(1, t) = 0$ and $u(x, 0) = \sin px, 0 \le x \le 1$ using the Schmidt method (Take h = 0.2 and $\alpha = 1/2$). (5)

11. Apply Milne's method, to find a solution of the differential equation $y' = x - y^2$ in the range $0 \le x \le 1$ for the boundary condition $y = 0$ at $x = 0$. (5)

(5)

12. Conclude the value of y and z at $x = 0.1$ by solving the simultaneous differential equations

$$\frac{dy}{dx} = 2y + z$$
$$\frac{dz}{dx} = y - 3z$$

Using Runge-Kutta method of fourth order, given that $y(0) = 0$, $z(0) = 0.5$.

OR

Deduce the solution of the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial t^2}$ subject to the (5)

conditions $u(x, 0) = \sin sin\, \pi x$, $0 \le x \le 1$, $u(0, t) = u(1, t) = 0$ Schmidt method. Carryout computations for two levels, taking $h = 1/3$, $k = 1/36$.

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