



## BRAINWARE UNIVERSITY

Term End Examination 2023  
Programme – M.Sc.(MATH)-2021  
Course Name – Coding Theory  
Course Code - MSCME404  
( Semester IV )

Full Marks : 60

Time : 2:30 Hours

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

### Group-A

(Multiple Choice Type Question)

1 x 15=15

1. Choose the correct alternative from the following :

(i) Choose the right option: In the ring  $Z$  of integers

- a) all integers form an ideal                      b) all the even integers form an ideal  
c) all the odd integers form an ideal              d) No integers form an ideal

(ii) For  $x, y \in F_q^n$  and  $d(x, y) = wt(x + y)$  then select the right option for  $q$

- a)  $q$  is even    b)  $q$  is odd  
c)  $q$  is zero    d)  $q$  is negative

(iii) Indicate  $p$  in a finite subfield  $F_p$

- a) Smallest positive integer                      b) prime number  
c) Characteristic of field  $F$                       d) All of these

(iv) Select the right option: A field with cardinality  $q$  is isomorphic to

- a)  $F_{q^n}$     b)  $F_{q^2}$   
c)  $F_q$     d) None of these

(v) Choose the correct option:  $F_q^n$  is a

- a) Abelian group but not finite                      b) finite abelian group  
c) Finite non-abelian group                          d) None of these

(vi) Select the right option: The field  $F_p$  isomorphic to

- a)  $Z/pZ$     b)  $Z/p^2Z$   
c)  $Z$     d)  $pZ$

(vii) Estimate the distance of a linear code is

- a) the largest weight of its nonzero codewords      b) the smallest weight of its nonzero codewords  
c) the smallest weight of two zero codewords      d) the largest weight of two zero codewords

- (viii) Choose the correct codes: The weight enumerator of which of the following two codes satisfy the McWilliams theorem?
- a) Reed Muller and Reed Solomon                      b) Kerdock and Preparata  
c) Constacyclic and Negacyclic                      d) Hadamard and Quadratic residue
- (ix) Select which of the following is not an error correcting codes?
- a) Binary Convolution codes                      b) Low Density Parity Check  
c) Reed-Solomon Codes                      d) None of these
- (x) In factorization of  $x^n - 1$ , for  $n=2$ , calculate the number of binary cyclic codes found
- a) 2                      b) 3  
c) 4                      d) 7
- (xi) For a (7,4) cyclic code, the generator polynomial  $g(x)=1+x+x^3$ , Select the data word of the codeword 0110100.
- a) 010                      b) 0001  
c) 0110                      d) 1011
- (xii) Identify the correct option: Hamming Code is
- a) Linear Block Codes                      b) Non Linear Block codes  
c) No binary code                      d) None of these
- (xiii) For a  $[n, k, d]$  linear code  $C$  for  $u, v$  are in same coset of  $C$  and syndrome  $S$ , conclude which one is true?
- a)  $S(u) > S(v)$                       b)  $S(u) = S(v)$   
c)  $S(u) < S(v)$                       d) none of these
- (xiv) Tell the minimum Hamming distance between any two correct code words.
- a) 1                      b) 2  
c) 3                      d) 4
- (xv) Select the correct option : A code  $C = \{(\lambda, \lambda, \dots, \lambda) : \lambda \in F_q\}$  is called
- a) Constant code                      b) Repetition code  
c) Cyclotomic code                      d) All of these

**Group-B**

(Short Answer Type Questions)

3 x 5=15

2. State the Minimum distance decoding rule. (3)
3. Solve all the primitive elements of the following fields:  $F_7, F_8$ . (3)
4. Describe extended code of a code  $C$  and parity-check coordinate. (3)
5. Justify the definition of perfect code. (3)

6. Select the cosets of the binary linear code  $C = \{0000, 1011, 0101, 1110\}$  (3)

Justify Sphere-covering bound. OR (3)

**Group-C**  
(Long Answer Type Questions)

5 x 6=30

7. Define Maximum likelihood decoding and discuss its types. (5)

8. Describe the properties of Binary Hamming code. (5)

9. Justify MacWilliams Theorem. (5)

10. If  $V$  be a vector space over  $F_q$  and  $\dim(V)=k$  then evaluate (5)  
that  $V$  has  $\frac{1}{k!} \prod_{i=0}^{k-1} (q^k - q^i)$  different bases.

11. For all integers  $r \geq 0$ , evaluate that a sphere of radius  $r$  in  $A^n$  (5)  
contains exactly  $V_q^n(r)$  vectors, where  $A$  is an alphabet of size  
 $q > 1$ .

12. For any two non-zero elements  $\alpha, \beta \in F_q^*$ , if  $\gcd(\text{ord}(\alpha),$  (5)  
 $\text{ord}(\beta))=1$  then conclude that  $\text{ord}(\alpha)$  divides  $q-1$  and  
 $\text{ord}(\alpha\beta)=\text{ord}(\alpha) \times \text{ord}(\beta)$ .

OR

Distinguish weight enumerator and distance enumerator of a code. (5)

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