



BRAINWARE UNIVERSITY

Term End Examination 2023
Programme – M.Sc.(MATH)-2021
Course Name – Stochastic Processes
Course Code - MSCME407
(Semester IV)

Full Marks : 60

Time : 2:30 Hours

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group-A

(Multiple Choice Type Question)

1 x 15=15

1. Choose the correct alternative from the following :

(i) Compute the variance of a negative binomial distribution with parameter r and p .

- a) $rp(1-p)$
- b) $r(1-p)/p^2$
- c) $rp^2(1-p)$
- d) $r^2p(1-p)$

(ii) Which of the following is known as 'memory less' property? Select the correct option.

- a) $P(X > s + t | X > t) = P(X > s), \forall s, t \geq 0$
- b) $P(X < s + t | X > t) = P(X > s), \forall s, t \geq 0$
- c) $P(X > s + t) = P(X > s), \forall s, t \geq 0$
- d) none of these

(iii) The probability of any event A satisfies _____. Select the correct option.

- a) $P(A) \geq 1$
- b) $P(A) < 0$
- c) $0 \leq P(A) \leq 1$
- d) None of these

(iv) The distribution function $F(x)$ of a random variable X is given by []. Select the correct option.

- a) $P(-\infty < X < \infty)$
- b) $P(-\infty < X \leq x)$
- c) $P(-\infty \leq X < \infty)$
- d) none of these.

(v) Let $X(t)$ and $Y(t)$ be two random processes with respective auto correlation functions

$R_{xx}(\tau)$ and $R_{yy}(\tau)$. Then compute $|R_{xy}(\tau)|$

- a) $= \sqrt{R_{xx}(0)R_{yy}(0)}$
- b) $\geq \sqrt{R_{xx}(0)R_{yy}(0)}$
- c) $\leq \sqrt{R_{xx}(0)R_{yy}(0)}$
- d) $> \sqrt{R_{xx}(0)R_{yy}(0)}$

Group-B
(Short Answer Type Questions)

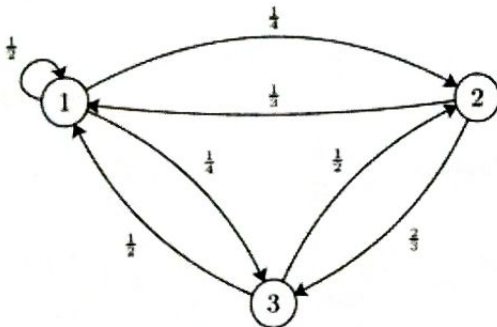
3 x 5=15

2. Describe discrete time stochastic processes. (3)
3. Consider the Markov chain with three states, $S = \{1,2,3\}$, that has the following transition matrix (3)

$$\begin{bmatrix} \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{3} & 0 & \frac{2}{3} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$$

Sketch the state transition diagram.

4. Consider the Markov chain shown in Figure (3)



Determine if this chain is reducible.

5. Consider the following transition matrix (3)

$$\begin{bmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

Justify if the matrix is valid TPM.

6. Consider the Markov chain with three states, $S = \{1,2,3\}$, that has the following transition matrix (3)

$$\begin{bmatrix} 0 & 0 & 1 \\ \frac{1}{3} & 0 & \frac{2}{3} \\ \frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix}$$

Determine if this chain is communicative.

OR

$$P = \begin{bmatrix} \frac{3}{4} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} \quad (3)$$

Consider the Markov chain with two states and transition probability matrix
Compute the stationary probabilities of the chain.

Group-C

(Long Answer Type Questions)

5 x 6=30

7. Summarize the concept of branching processes with an example. (5)
8. Summarize the concept of birth and death processes with examples. (5)
9. Explain the concept of transition probability matrix with an example. (5)
10. Summarize the transition probability diagram in DTMC. (5)
11. Establish the concepts of merging independent Bernoulli processes with examples (5)
12. Summarize the concept of Periodic and aperiodic states in DTMC. (5)

OR

Summarize the concept of Recurrent and transient states in DTMC. (5)
