BRAINWARE UNIVERSITY

Term End Examination 2023-2024
Programme – B.Sc.(BT)-Hons-2020/B.Sc.(BT)-Hons-2021
Course Name – Bio-Mathematics
Course Code - BBTD502C
(Semester V)

Full Marks : 60 **Time : 2:30 Hours**

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group-A

(Multiple Choice Type Question) 1 x 15=15

1. *Choose the correct alternative from the following :*

(i) Identify the correct answer, the population doubling time for Logistically growing species is

a) $T = \frac{1}{r}\ln\left[\frac{2K-2N_0}{K-2N_0}\right]$ b) $T = \ln\left[\frac{2K-2N_0}{K-2N_0}\right]$

c) $T = \frac{1}{r}\ln\left[\frac{2K-2N_0}{2N_0}\right]$ d) none of these.

(ii) Choose the correct option,

$$\frac{dx}{dt} = x(\alpha - \beta y)$$
$$\frac{dy}{dt} = y(-\gamma + \delta x)$$

The name of the model knows

a) Logistic b) Lotka-Voltera model

c) Gompertz d) None of these

(iii) Consider the autonomous nonlinear system

$$\dot{x} = -2x + 3y + xy$$
$$\dot{y} = -x + y - 2xy^2$$

The characteristic equation respect to the equilibrium point (0,0) is, Choose the correct option.

a) $\lambda^2 + 1 = 0$ b) $\lambda^2 + \lambda + 1 = 0$

c) $\lambda^2 + 2\lambda + 1 = 0$

d) none of these

(iv) Consider the autonomous nonlinear system

$$\dot{x} = -2x + 3y + xy$$
$$\dot{y} = -x + y - 2xy^2$$

The nature of the equilibrium point (0,0) is. Choose the correct option

a) Nonhyperbolic

b) Hyperbolic

c) Both (a) and (b)

d) None of these

(v) Consider the autonomous nonlinear system

$$\dot{x} = -2x + 3y + xy$$
$$\dot{y} = -x + y - 2xy^2$$

The nature of the equilibrium point (0,0) is, Choose the correct option

a) Saddle

b) Stable node

c) Stable focus

d) None of these

(vi)

$$\frac{dx}{dt} = x(1 - x) - xy$$

$$\frac{dy}{dt} = \beta y(x - \alpha)$$

The number of critical points occurs for the given system, Choose the correct option

a) 4

b) 3

c) 2

d) None of these

(vii) The system represented as $\frac{dB}{dT} = rB\left(1 - \frac{B}{K}\right) - \frac{mB^2}{a^2+B^2}$ for Nondimensionalization we apply $B = xa$ and $T = \frac{a}{m}t$ then the system becomes $\frac{dx}{dt} = px\left(1 - \frac{x}{q}\right) - \frac{x^2}{1+x^2}$ then the value of p is, Choose the correct option.

a) $\frac{ar}{m}$

b) $\frac{a}{m}$

c) $\frac{r}{m}$

d) None of these

(viii) Choose the correct option, $x_{t+1} = f(x_t)$ be the discrete model, f' is continuous and $\bar{x}$ be the equilibrium point of then system is unstable if

a) $|f'(\bar{x})| > 1$

b) $|f'(\bar{x})| < 1$

c) $|f'(\bar{x})| = 1$

d) None of these

(ix) The Density dependence growth model may be express in the form of……., N(t) consider be the density of the species at time t

a) $\frac{dN}{dt} = \mathrm{rN} - \mathrm{a}N^2$

b) $\frac{dN}{dt} = \mathrm{r}$

c) $\frac{dN}{dt} = -\mathrm{a}$

d) none of these.

(x) If we express the system $\frac{dN}{dt} = \mathrm{rN} - \mathrm{a}N^2$ in the form of $\frac{dN}{dt} = \mathrm{rN}\left(1 - \frac{N}{K}\right)$ then K is

a) $K = r$ b) $K = a$

c) $K = \frac{r}{a}$ d) none of these.

(xi) $\frac{dN}{dt} = \mathrm{rN}\left(1 - \frac{N}{K}\right)$, where $N =$ *Population density*, $r =$ *intrinsic growth rate*, $K =$ *carrying capacity*. The given model is classified as

a) Logistic growth model. b) Malthus growth model.

c) Two species model. d) none of these.

(xii) Identify the critical point of the Logistic growth model $\frac{dN}{dt} = \mathrm{rN}\left(1 - \frac{N}{K}\right)$ from the given option.

a) K b) 1

c) 2 d) None of these

(xiii) Identify the critical point of the Density dependent growth model $\frac{dN}{dt} = \mathrm{rN} - aN^2$ from the given option.

a) 1 b) $\frac{r}{a}$

c) 2 d) None of these

(xiv) Identify the correct answer, the population doubling time for the exponentially growing species is

a) $\ln 2$ b) $\frac{\ln 2}{r}$

c) r d) None of these

(xv) Identify the correct answer, the point of inflexion of logistic growth model is

a) $N = K$ b) $N = \frac{K}{3}$

c) $N = \frac{K}{2}$ d) None of these

Group-B

(Short Answer Type Questions) 3 x 5=15

2. Compute that the model is unstable around the trivial equilibrium point: (3)

$$\frac{dx}{dt} = x(1 - x) - xy$$

$$\frac{dy}{dt} = \beta y(x - \alpha)$$

Where α *and* β *are positive constant.*

3. Describe exponential growth model. (3)

4. Explain Bifurcation. (3)

5. Describe Allee Effect. (3)

6. Explain Asymptotic stability. (3)

OR

Explain Hyperbolic equilibrium point, give an example. (3)

Group-C

(Long Answer Type Questions) 5 x 6=30

7. Describe Exponential growth model with graphical representation. (5)

8. Explain the stability of the equilibrium point (1,0) and draw the phase plane: (5)

$$\frac{dx}{dt} = x(1-x) - xy$$

$$\frac{dy}{dt} = \beta y(x-\alpha)$$

Where α *and* β *are positive constant.*

9. Deduce the stability condition of the Predator free equilibrium point of a predator prey model type-II, and draw the phase portrait. (5)

$$\frac{dN}{dT} = rN\left(1-\frac{N}{K}\right) - \frac{CNP}{a+N}$$

$$\frac{dP}{dT} = \frac{bNP}{a+N} - mP$$

10. Illustrate Bendixson's Theorem and prove it. (5)

11. Deduce the two prey one predator model of holling type-II model. (5)

12. Evaluate the equilibrium point of the discrete predator-prey model: (5)

$$x_{t+1} = x_t(a - x_t - y_t), a > 0$$

$$y_{t+1} = y_t(b + x_t), 0 < b < 1$$

Discuss the stability around $(a-1,0)$.

OR

$$S_{t+1} = S_t - \frac{\beta}{N} S_t I_t + b(N - S_t) \quad (5)$$

$$I_{t+1} = I_t(1 - \gamma - b) + \frac{\beta}{N} S_t I_t$$

Where S_t *and* I_t be the susceptible and infected individuals at time t. Evaluate the Basic reproduction number($\Re_0$) about the disease-free equilibrium point?

****