BRAINWARE UNIVERSITY

Term End Examination 2023-2024
Programme – M.Sc.(MATH)-2023
Course Name – Fuzzy Logic
Course Code - MSCMC204
(Semester II)

Full Marks : 60 **Time : 2:30 Hours**

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group-A

(Multiple Choice Type Question) 1 x 15=15

1. *Choose the correct alternative from the following :*

(i) Tell the name of the founding father of Artificial Intelligence:

a) Warren McCulloch
b) Walter Pitts
c) Lotfi A. Zadeh
d) John McCarthy

(ii) For a left right membership function specified by LR(x;p,q,r), tell correct statement:

a) At p, the membership value is 1.
b) The width of left region varies with q.
c) The width of right region varies with r.
d) All of these

(iii) Identify true statement for the law of contradiction of crisp set A.

a) $A \cup \bar{A} = A$
b) $A \cup \bar{A} = \phi$
c) $A \cap \bar{A} = \phi$
d) None of these

(iv) Using De Morgan's Law, identify the correct statements for crisp sets X and Y:

a) $\overline{X \cup Y} = \bar{X} \cap \bar{Y}$
b) $\overline{X \cup Y} = \bar{X} \cup \bar{Y}$
c) $\overline{X \cap Y} = \bar{X} \cap \bar{Y}$
d) None of these

(v) For the fuzzy sets R and S defined below:

R = { (10,0.7), (30,1.0) }

S = { (13,0.9), (15,0.5) }

Identify the membership value at $\delta\delta = 15$ for the distance set between R and S:

a) 0.1

b) 0.5

c) 0.9

d) 0.7

(vi) Let A and B are two fuzzy sets given as below.

$A = 0.8/1 + 0.4/2$

$B = 0.5/1 + 0.2/2$

Determine the intersection of fuzzy sets A and B for the universe of discourse X={1,2} using Dombi's class of T-norm for $\lambda = 1$

a) 0.78/1 + 0.90/2

b) 0.44/1 + 0.15/2

c) 0.15/1 + 0.44/2

d) 0.15/1 + 0.50/2

(vii) Let A and B are two fuzzy sets defined over the universe of discourse X, examine the formulation for S-norm bounded sum operator:

a) $1 \vee (\mu_A(x) + \mu_B(x))$

b) $1 \wedge (\mu_A(x) - \mu_B(x))$

c) $1 \wedge (\mu_A(x) + \mu_B(x))$

d) $1 \vee (\mu_A(x) - \mu_B(x))$

(viii) Identify the law of contradiction for fuzzy relations:

a) $R \cap \bar{R} = O$

b) $R \cap \bar{R} \neq O$

c) $R \cup \bar{R} = O$

d) $R \cup \bar{R} \neq O$

(ix) Let us consider fuzzy sets B and C with the universe of discourse Y and Z, respectively defined as

$B = 0.1/y1 + 0.5/y2 + 0.9/y3$

$C = 0.1/z1 + 0.3/z2 + 0.8/z3$

Identify the diagonal values of fuzzy relation matrix S between B and C:

a) 0.1, 0.3, 0.8

b) 0.1, 0.3, 0.7

c) 0.9, 0.7, 0.7

d) 0.9, 0.7, 0.3

(x) Let fuzzy set B is defined with the universe of discourse X = {1,2,3} given as below.

B = 0.5/1 + 1.0/2 + 0.2/3

Determine the correct option that will be the fuzzy set obtained by $INT(B)$:

a) 0.50/1 + 0.00/2 + 0.20/3 b) 0.50/1 + 0.00/2 + 0.08/3

c) 0.50/1 + 1.00/2 + 0.20/3 d) 0.50/1 + 1.00/2 + 0.08/3

(xi) Consider a fuzzy set A defined on the interval X = [0, 10] of integers by the membership function $\mu_A(x) = x / (x+2)$. Evaluate the α-cut corresponding to α= 0.5

a) {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10} b) {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

c) {2, 3, 4, 5, 6, 7, 8, 9, 10} d) None of these

(xii) If A and B are two fuzzy sets with membership functions: $\mu_a(\chi)$ ={0.2,0.5.,0.6,0.1,0.9}

$\mu_b(\chi)$= {0.1,0.5,0.2,0.7,0.8} then evaluate the value of $\mu_a \cap \mu_b$

a) {0.2,0.5,0.6,0.7,0.9} b) {0.2, 0.5,0.2, 0.1,0.8}

c) {0.1, 0.5, 0.6, 0.1,0.8} d) {0.1, 0.5, 0.2, 0.1,0.8}

(xiii) Choose the correct option. The height h(A) of a fuzzy set A is defined as h(A) = sup A(x). A is normal if:

a) h(A) = 0 b) h(A) <0

c) h(A)=1 d) h(A)<1

(xiv) Suppose the function y and a fuzzy integer number around -4 for x are given as y= (x-3)2 + 2. Around -4 = {(2, 0.3), (3, 0.6), (4, 1), (5, 0.6), (6, 0.3)} respectively. Then evaluate f (Around -4) is given by

a) {(2, 0.6), (3, 0.3), (6, 1), (11, 0.3)} b) {(2, 0.6), (3, 1), (6, 1), (11, 0.3)}

c) {(2, 0.6), (3, 1), (6, 0.6), (11, 0.3)} d) {(2, 0.6), (3, 0.3), (6, 0.6), (11, 0.3)}

(xv) Choose the correct option. Let Aα0 denotes the α-cut of a fuzzy set A at α0. If α1< α2 ,then

a) $A\alpha1 \supseteq A\alpha2$ b) $A\alpha1 \supset A\alpha2$

c) $A\alpha1 \subseteq A\alpha2$ d) $A\alpha1 \subset A\alpha2$

Group-B

(Short Answer Type Questions) 3 x 5=15

2. Calculate using arithmetic operations: (3)

i) $\frac{[4,10]}{[1,2]}$,

ii) $[3,4]\cdot[2,2]$,

iii) $\frac{[-1,1]}{[-2,-\frac{1}{2}]}$

3. Define scalar cardinality of a fuzzy set with an example. (3)

4. Define standard Fuzzy intersection and union. (3)

5. Describe equilibrium point of a fuzzy complement with example. (3)

6. If a fuzzy complement C is defined by, $C(a) = \frac{1-a}{1+\lambda a}, \lambda \geq 0$ evaluate the equilibrium point. (3)

OR

Let, $g_w : [0,1] \rightarrow \mathbb{R}$ defined by, $g_w(a) = a^w$, then (3)

$$g_w^{(-1)}(a) = \begin{cases} 0 & \text{if } a \in (-\infty, 0) \\ a^{\frac{1}{w}} & \text{if } a \in [0,1] \\ 1 & \text{if } a \in (1, \infty) \end{cases}$$

evaluate the fuzzy complement C_λ generated by g_λ.

Group-C

(Long Answer Type Questions) 5 x 6=30

7. Discuss Weighted Average Method with example. (5)

8. For a Fuzzy sets A and B defined on $X = \{x_1, x_2, x_3, x_4, x_5\}$ by, (5)

$$A = \frac{0.1}{x_1} + \frac{0.7}{x_3} + \frac{0.9}{x_4} + \frac{1}{x_5} \qquad B = \frac{0.3}{x_1} + \frac{0.1}{x_2} + \frac{0.6}{x_3} + \frac{1}{x_4} + \frac{0.5}{x_5}$$

Evaluate $\bar{A}, \bar{B}, A \cup B, A \cap B, \overline{A \cup B}$.

9. Let R be a fuzzy relation defined by the following relational matrix (5)

$$R = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{cccccc} y_1 & y_2 & y_3 & y_4 & y_5 & y_6 \end{array} \\ \begin{bmatrix} 0.1 & 0.2 & 0.4 & 0.8 & 1 & 0.8 \\ 0.2 & 0.4 & 0.8 & 1 & 0.8 & 0.6 \\ 0.4 & 0.8 & 1 & 0.8 & 0.4 & 0.2 \end{bmatrix} \end{array}$$

Evaluate the projection of $R(x, y)$ on X and Y.

10. Justify that Yager's class of fuzzy complements is continous and involutive. (5)

11. Write the definition of aggregation operation on n fuzzy sets. (5)

12. Justify that a fuzzy set A on $\mathbb{R}$ is convex if (5)

$$A(\lambda x_1 + (1-\lambda)x_2) \geq \min(A(x_1), A(x_2)), \forall x_1, x_2 \in \mathbb{R}$$

OR

Justify that, for any Fuzzy sets A and B defined on X, $A \cap (A \cup B) = A$. (5)

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