BRAINWARE UNIVERSITY

Term End Examination 2023-2024
Programme – M.Sc.(MATH)-2023
Course Name – General Topology
Course Code - MSCMC205
(Semester II)

Full Marks : 60 **Time : 2:30 Hours**

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group-A

(Multiple Choice Type Question) 1 x 15=15

1. *Choose the correct alternative from the following :*

(i) Let $X=\{a,b,c,d\}$. Then select the collections of subsets that is a topological structure on X.

a) $\{\phi, X, \{a,b\}, \{c\}\}$
b) $\{\phi, X, \{a,b\}, \{b,c\}\}$
c) $\{\phi, X, \{a,b\}, \{c,d\}\}$
d) $\{\phi, X, \{a,b\}, \{c\}, \{d\}\}$

(ii) Select the interval from the followings that generates lower limit topology.

a) (a, b)
b) [a, b)
c) (a, b]
d) [a, b]

(iii) Let (X, τ) be a topological space. Then select the values of ϕ^o and X^o respectively.

a) X and ϕ
b) ϕ and X
c) X and X
d) ϕ and ϕ

(iv) Identify the correct option. Under indiscrete topology on $\mathbb{R}$, the set of natural numbers is

a) Open
b) Both open and closed
c) Closed
d) None of these

(v) Identify the Closure set of the set **RQ** of all irrational number is

a) **Q**
b) **R**
c) **RQ**
d) Empty set

(vi) Select the correct option. Every finite subset of a metric space X is

a) open
b) closed
c) both open and closed
d) neither open nor closed

(vii) Identify the boundary of (0,1)

a) (0, 1)
b) {0,1}
c) {0}
d) {1}

(viii) Choose the correct option: Let $X = \{a, b, c\}$ and $T = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$. Then T is

a) the indiscrete topology on X
b) the discrete topology on X
c) a non-Hausdorff topology on X
d) a Hausdorff topology on X

(ix) Choose the correct option: the diameter of a subset A of a metric space (X, d) is

a) $Sup\{d(x,y)|(x,y) \in A\}$
b) $Inf\{d(x,y)|(x,y) \in A\}$
c) $Max\{d(x,y)|(x,y) \in A\}$
d) None of these

(x) Choose the correct option: A discrete topological space with at least 2 points is

a) Disconnected
b) Bounded
c) Compact
d) None of these

(xi) Select the correct option: A regular T_1 space is always-

a) T_1 but not T_2
b) T_2 but not T_1
c) may not be T_1 or T_2
d) both T_1 and T_2

(xii) Select the correct option: A metric space is

a) T_1 but not normal
b) normal but not T_1
c) both T_1 and normal
d) neither normal nor T_1

(xiii) Select the correct option: Suppose $f : X \longrightarrow Y$ is continuous and onto then, if X is path-connected, then Y is

a) path connected
b) compact
c) connected
d) None of these

(xiv) Select the correct option: Every finite T_1 space is a

a) indiscrete space
b) discrete space
c) No such space exists
d) Depends on the structure

(xv) Select the correct option: Urysohn lemma applicable on the

a) compact space
b) connected space
c) normal space
d) regular space

Group-B

(Short Answer Type Questions) 3 x 5=15

2. Examine, whether the union $T_1 \cup T_2$ of two topologies on a set X is a topology on X or not. (3)

3. Describe the pasting lemma. (3)

4. Write all the open sets in a discrete metric space. (3)

5. Analyze that $\{x \in \mathbb{Q}: -1 < x < 1\}$ is open in $\mathbb{Q}$ but not closed in $\mathbb{Q}$. (3)

6. Justify by an example that a first countable space may be neither separable nor Lindeloff. (3)

OR

Justify that a metric space (X, d) is 1^{st} countable. (3)

Group-C

(Long Answer Type Questions) 5 x 6=30

7. Let $f: (X, \tau) \to (Y, \sigma)$ be a map. Then show that f is continuous if and only if for each $x \in X$ and any neighbourhood W of $f(x)$ in Y, there exists a neighbourhood V of x such that $f(V) \subset W$. (5)

8. Illustrate that compactness is a topological property. (5)

9. Let $f: A \to B$ and $g: C \to D$ be continuous functions. Define a map $f \times g: A \times C \to B \times D$ by the equation $(f \times g)(a, c) = f(a) \times g(c)$. Deduce that $f \times g$ is continuous. (5)

10. For a subset A of a space X, examine that $\bar{A} = A \cup A'$. (5)

11. Justify that separability is a topological property. (5)

12. Justify that a closed subspace of a normal space is normal. (5)

OR

Justify that a compact Hausdorff space is normal. (5)

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