BRAINWARE UNIVERSITY

Term End Examination 2023-2024
Programme – M.Sc.(MATH)-2023
Course Name – Complex Analysis
Course Code - MSCMC202
(Semester II)

Full Marks : 60 **Time : 2:30 Hours**

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group-A
(Multiple Choice Type Question) 1 x 15=15

1. *Choose the correct alternative from the following :*

(i) Choose the correct property is common to all the following functions. $f_1(z) = \bar{z}, f_2(z) = \mathrm{Re}(z), f_3(z) = \mathrm{Im}(z), f_4(z) = 2 + \mathrm{Re}(z)$

a) All the four functions are real valued
b) All the functions are bounded
c) All the functions are continuous everywhere but differentiable nowhere
d) All are analytic

(ii) Locate the region where the function $f(z) = |z|$ is analytic.

a) Everywhere
b) Nowhere
c) Only at z=0
d) Everywhere except at z=0

(iii) Identify the correct option-The function $f(z) = |z|^2$

a) Continuous nowhere
b) Continuous everywhere but no-where differentiable
c) Continuous everywhere but no-where differentiable except the origin
d) Continuous at origin only

(iv) Compute $\int_C \frac{1}{z} dz$, where $C: |z - 2| = 1$.

a) 0
b) $2\pi i$
c) 1
d) Can't be evaluated since the integral does not exist

(v) A circle on the Riemann sphere passing through the north pole describes

a) A straight line in the complex plane
b) A circle in the complex plane
c) An ellipse in the complex plane
d) A line-segment in the complex plane

(vi) Let f be an entire function such that $f(iy) = exp\ (iy)\ , 0 \leq y \leq 1$. Then write the correct option.

a) $f(x + iy) = exp\ (x + iy)$, for every x and y
b) $f(x + iy) = exp\ (iy)$

c) $f(x+iy) = exp\,(x+iy)$ for every x and $0 \le y \le 1$ d) None of these

(vii) The transformation $w = e^{i\theta}\left(\frac{z-p}{\bar{p}z-1}\right)$, where p is a constant, maps $|z| < 1$ onto

a) $|w| < 1\ if\ |p| < 1$
b) $|w| < 1\ if\ |p| > 1$
c) $|w| < 1\ if\ |p| = 1$
d) $|w| = 3\ if\ p = 0$

(viii) Evaluate the residue of $\frac{z^2}{(z^2+1)^2}$ at $z = \infty$.

a) 1
b) 0
c) -1
d) 2

(ix) Identify residue of $\frac{\cos z}{z}$ at z=0 is

a) 1
b) -1
c) 2
d) 0

(x) Evaluate the value of $\frac{1}{2\pi i}\oint_{|z|=4} \frac{e^z}{(z+2)^2}dz$.

a) e^3
b) e^2
c) 0
d) None of these

(xi) Choose the correct option-A non-constant entire function

a) Cannot have uncountable number of zeroes in C.
b) Cannot have a countable number of zeroes in a bounded region.
c) Cannot have three zeroes lying on a straight line
d) Should have at least one zero in C.

(xii) Let $f(z) = cos\frac{1}{z}$, then choose the correct option.

a) 0 is an isolated singularity of f(z)
b) 0 is a non-isolated singularity of f(z)
c) 0 is an isolated essential singularity of f(z)
d) 0 is a non-isolated essential singularity of f(z)

(xiii) Let $f(z) = \frac{e^z}{z^3}$ if $z \neq 0$. Evaluate the residue of f(z) at z=0.

a) 0
b) 1
c) 2πi
d) 1/2

(xiv) A Mobius transformation which transform the upper half plane into the lower half is

a) $w = \bar{z}$
b) $w = \frac{z-1}{z+i}$
c) $w = \frac{1}{z}$
d) $w = \frac{z+i}{z-i}$

(xv) Write the correct option-The function $w(z) = -\left(\frac{1}{z} + bz\right), 1 < b < 1, maps\ |z| < 1$ onto

a) A half plane
b) Exterior of a circle
c) Exterior of an ellipse
d) Interior of an ellipse

Group-B

(Short Answer Type Questions) 3 x 5=15

2. Illustrate the bilinear transformation which maps the points $z = 1, 0, -1$ into the points $w = i, 0, -i$. (3)

3. Compute the value of k so that the function $k\log(x^2 + y^2)$ is harmonic. (3)

4. Write Liouville's theorem and prove it. (3)

5. Using Cauchy's residue theorem explain that the singularities of $f(z) = \cot \pi z$, are all simple poles (3)

6. Justify that the maximum modulus of e^z is always assumed on the boundary of the compact domain. (3)

OR

Justify that zeros are isolated points. (3)

Group-C

(Long Answer Type Questions) 5 x 6=30

7. Explain that the transformation $w = \frac{z+i}{z-i}$ maps the interior of the circle $|w| = 1, i.e., |w| \leq 1$, into the lower half plane $I(z) \leq 0$. (5)

8. Evaluate $\int_0^{2\pi} \frac{d\theta}{1+0.5\sin\theta}$, using contour integration method. (5)

9. Test that if w=f(z) is an analytic function of z, $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) \log|f'(z)| = 0$. (5)

10. Write fundamental theorem of algebra and prove it. (5)

11. Explain log(1+z) in power of z and indicate the region of convergence. (5)

12. Justify that the function $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$, satisfies Laplace's equation and determine the corresponding analytic function. (5)

OR

Justify that any open disk in complex plane is an open set. Also justify that every closed disk in complex plane is a closed set. (5)

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