BRAINWARE UNIVERSITY

Term End Examination 2023-2024
Programme – M.Sc.(MATH)-2023
Course Name – Partial Differential Equations
Course Code - MSCMC203
(Semester II)

Full Marks : 60 **Time : 2:30 Hours**

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group-A
(Multiple Choice Type Question) 1 x 15=15

1. *Choose the correct alternative from the following :*

(i) Tell the solution of the differential equation $r + 5s + 6t = (y - 2x)^{-1}$

a) $\phi_1(y + 2x) + \phi_2(y + 3x) + log\ (y + 2x)$
b) $\phi_1(y - 2x) + \phi_2(y + 3x) + xlog(y - 2x)$
c) $\phi_1(y - 2x) + \phi_2(y - 3x) + xlog(y + 2x)$
d) $\phi_1(y - 2x) + \phi_2(y - 3x) + xlog(y - 2x)$

(ii) Select the region in which the following differential equation is hyperbolic.

$$yu_{xx} + 2xyu_{xy} + xu_{yy} = u_x + u_y$$

a) $xy \neq 1$
b) $xy \neq 0$
c) $xy > 1$
d) $xy > 0$

(iii) Choose the correct option. The singular solution of the differential equation $(xp - y^2) = p^2 - 1$ is

a) $x^2 + y^2 = 1$
b) $y^2 - x^2 = 1$
c) $x^2 + 2y^2 = 1$
d) $x^2 - y^2 = 1$

(iv) Select the correct option. The solution of the given differential equation $\frac{\partial^4 z}{\partial x^4} - \frac{\partial^4 z}{\partial y^4} = 0$, is

a) $f_1(y + x) + f_2(y - x) + f_3(y + ix) + f_4(y - ix)$
b) $f_1(y + x) + f_2(y - x)$
c) $f_1(y + ix) + f_2(y - ix)$
d) None of these

(v) Select the correct option. Monge's method is used to solve a partial differential equation of

a) n^{th} order
b) 1^{st} order
c) 2^{nd} order
d) None of these

(vi) Determine the complete solution of

$z = px + qy + p^2 + q^2$

a) $z = ax + by + a^2 + b^2$
b) $z = ax + by$
c) $z = a^2x^2 + b^2y^2$
d) None of these

(vii) Classify the one-dimensional wave equation

a) $\frac{\partial^2 u}{\partial y^2} = \frac{1}{c^2} \cdot \frac{\partial^2 u}{\partial x^2}$
b) $\frac{\partial^2 u}{\partial t^2} = \frac{1}{c^2} \cdot \frac{\partial^2 u}{\partial x^2}$
c) $\frac{\partial^2 u}{\partial t^2} = c^2 \cdot \frac{\partial^2 u}{\partial x^2}$
d) $\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2} \cdot \frac{\partial^2 u}{\partial t^2}$

(viii) Select the correct option. In the diffusion equation $\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$, $\alpha =$

a) $\alpha = K/\rho C$
b) $\alpha = K \rho C$
c) $\alpha = \rho C/K$
d) None of these

(ix) Select the correct option. Solution of $pt - qs = q^3$ is

a) $y = xz + f(z) + g(z)$
b) $y = xz + f(x) + g(z)$
c) $y = xz + f(x) + g(x)$
d) None of these

(x) Select the correct option. The equation of the envelope of surface represented by complete integral of the given PDE is called

a) Singular solution
b) Particular Integral
c) General integral
d) None of these

(xi) Choose the correct option. The solution of the PDE $y^2p - xyq = x(z - 2y)$ is

a) $\phi(x^2 + y^2, zy - y^2) = 0$, where ϕ being an arbitrary function.
b) $\phi(x^2 - y^2, zy - y^2) = 0$, where ϕ being an arbitrary function.
c) $\phi(x^2 + y^2, zy + y^2) = 0$, where ϕ being an arbitrary function.
d) None of these

(xii) In two-dimension heat flow, categorize the temperature along the normal to the xy-plane

a) zero
b) infinity
c) finite
d) 100K

(xiii) Select the correct option. The general form of 3-dimensional Heat equation is

a) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = k^2 \frac{\partial u}{\partial t}$
b) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = \frac{1}{k^2} \frac{\partial u}{\partial t}$
c) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = \frac{1}{k} \frac{\partial u}{\partial t}$
d) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = \frac{\partial u}{\partial t}$

(xiv) Choose the correct option for which one of the following is potential equation.

a) Heat equation b) Wave equation c) Laplace equation d) None of these

(xv) Choose the correct option. Solution of Heat equation when one end is insulated, then another end will be

a) Constant temperature b) Variable temperature

c) Initial temperature d) None of these.

Group-B

(Short Answer Type Questions) 3 x 5=15

2. Illustrate Stability theorem for Laplace equation. (3)

3. Explain the Monge's method for solving a PDE. (3)

4. Explain Separation of Variables Method for the wave equation $u_{tt} - c^2u_{xx} = 0, 0 \leq x \leq L, t > 0$ subject to boundary conditions $u(0,t) = 0, u(L,t) = 0, t > 0$ and initial conditions $u(x,0) = f(x), u_t(x,0) = g(x)$ for the separation constant $k > 0$. (3)

5. Identify a partial differential equation by eliminating a and b from the equation (3)

$z = ax + by + a^2 + b^2$.

6. Develop the solution of the heat equation using separation of variable method. (3)

OR

Apply the method of separation of variables to solve the following problem: (3)

$\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2}$ satisfying the following conditions

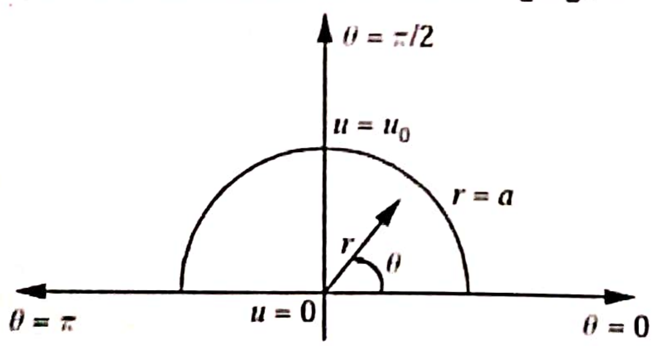
(i) $T = 0$ when $x = 0$ and 1 for all t

(ii) $T = \{2x, \quad 0 \leq x \leq \frac{1}{2}\ 2(1-x), \quad \frac{1}{2} \leq x \leq 1\ \ when\ t = 0$

Group-C

(Long Answer Type Questions) 5 x 6=30

7. Illustrate the steady state temperature in a semi-circular plate of radius a, insulated on both the faces with its curved boundary kept at a constant temperature U0 and its bounding diameter kept at zero temperature as described in the following figure (5)



8. Examine the following problem: (5)

$$u_{tt} - c^2 u_{xx} = 0, \qquad 0 < x < L, t > 0$$

Subject to the conditions: $u(x,0) = g(x), u(0, t) = 0, u(L,t) = 0, u_t(x,0) = h(x), 0 < x < L, t \geq 0.$

9. Deduce the two-dimensional Laplace equation in Cartesian coordinates using the method separation of variables. (5)

10. A uniform rod of length L whose surface is thermally insulted is initially at temperature $\theta = \theta_0$. At time $t = 0$, one end is suddenly cooled to $\theta = 0$ and subsequently maintained at this temperature; the other end remains thermally insulted. Evaluate the temperature distributed $\theta(x,t)$. (5)

11. Choose the separation of variables method and solve the following Helmholtz equation, using $\nabla^2 u + k^2 u = u_{xx} + u_{yy} + u_{zz} + k^2 u = 0$. (5)

12. Test that the canonical form of the PDE $(1 + x^2)u_{xx} + (1 + y^2)u_{yy} + xu_x + yu_y = 0$ is $u_{\alpha\alpha} + u_{\beta\beta} = 0$. (5)

OR

Justify that the complete integral of $x^2p^2 + y^2q^2 - 4 = 0$ using Charpit's method is $z = a \ln x + \sqrt{4 - a^2} \ln y + b$. (5)

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