



BRAINWARE UNIVERSITY

Term End Examination 2023-2024

Programme – M.Tech.(CSE)-AIML-2022/M.Tech.(CSE)-AIML-2023

Course Name – Mathematics-II

Course Code - BSC-MMM201

(Semester II)

Full Marks : 60

Time : 2:30 Hours

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group-A

(Multiple Choice Type Question)

1 x 15=15

1. Choose the correct alternative from the following :

(i) Identify which of the following functions is not analytic.

a) $f(z) = z^6$

b) $f(z) = \frac{1}{z}, z \neq 0$

c) $f(z) = \log \log r + i\theta$

d) $f(z) = \frac{1}{(z-1)^3}$

(ii) Select the value of the integral $\int_C \frac{dz}{z^2+9}$, where C is the ellipse $\frac{x^2}{4} + \frac{y^2}{1} = 1$ and the integral is taken in positive sense.

a) 0

b) $2\pi i$

c) $-\pi i$

d) πi

(iii) Select the region where the function $f(z) = |z|$ is analytic.

a) Everywhere

b) Nowhere

c) Only at $z=0$

d) Everywhere except at $z=0$

(iv) Select the value of the integral $\int_{|z|=2} \frac{\cos z}{z^3} dz$,

a) πi

b) $-\pi i$

c) $2\pi i$

d) $-2\pi i$

(v) Let h be the finite difference. Then the forward difference operator is explained as _____. Choose the correct option.

- a) $f(x)=f(x+h)-f(x)$
 b) $f(x)=f(x-h)-f(x)$
 c) $f(x)=f(x \cdot h)$
 d) None of these

(vi) If $f(x) = 0$ has a root between a & b , then examine $f(a)$ & $f(b)$ are of _____ signs.

- a) Opposite
 b) Same
 c) Positive
 d) Negative

(vii) Newton-Raphson method is also interpreted as

- a) normal method
 b) tangent method
 c) parallel method
 d) None of these

(viii) Choose the appropriate option. Bisection method is

- a) conditionally and surely convergent
 b) unconditionally and surely convergent
 c) conditionally convergent
 d) None of these

(ix) If $f(x, y) = x^2 y^2$, then solve $df =$

- a) $2x^2 dx + dy$
 b) $dx - 2dy$
 c) $dx + dy$
 d) $2xy^2 dx + 2yx^2 dy$

(x) Euler Lagrange equation formula is written as

- a) $F_y - \frac{d}{dx}(F'_y) = 0$
 b) $F_y - \frac{d}{dx}(F''_y) = 0$
 c) $F_x - \frac{d}{dx}(F'_y) = 0$
 d) None of these

(xi) Select the correct option-which of the following is the linear transformation from R^3 to R^2 ?

(i) $f \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ x+y \end{pmatrix}$

(ii) $g \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} xy \\ x+y \end{pmatrix}$

(iii) $h \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} z-x \\ x+y \end{pmatrix}$

- a) only f
 b) only g
 c) only h
 d) all the transformations f, g, h

(xii)

$$A = \begin{pmatrix} 4 & 1 & -3 \\ 0 & -1 & 1 \\ 0 & 0 & 7 \end{pmatrix}$$

Select the correct option-A matrix is

- a) a lower triangular matrix
 b) an upper triangular matrix
 c) a diagonal matrix
 d) a scalar matrix

(xiii)

The sum of eigenvalues of $\begin{pmatrix} -1 & -2 & -1 \\ -2 & 3 & 2 \\ -1 & 2 & -3 \end{pmatrix}$ is estimated as

- a) 1
 b) 3
 c) -1
 d) -3

(xiv) Select the correct option. If $A^2=A$, then its Eigenvalues are either

- a) 0 or 2
 b) 1 or 2
 c) 0 or 1
 d) Only 0

(xv) Select-If A is a non-null square matrix then $A+A^T$ is a

- a) symmetric matrix
 b) null matrix
 c) Identity matrix
 d) None of these

Group-B

(Short Answer Type Questions)

3 x 5=15

2. $\frac{dy}{dx} = x^3 + y, y(0) = 1, h = 0.01$ then using Euler's formula compute the value of $y(0.02)$. (3)
3. Illustrate the value of $\oint e^z dz$ around the circle $|z| = 1$. (3)
4. Using Cauchy's residue theorem determine the singularities of $f(z) = \cot z$, are all simple poles and the residues of $f(z) = \cot z$ at all simple poles are 1. (3)
5. A linear mapping $T: \mathbb{R}^3 \rightarrow \mathbb{R}^4$ is defined by $T(x, y, z) = (y+z, z+x, x+y, x+y+z)$. Evaluate $\text{Ker } T$. (3)
6. Justify the expression of Newton's forward interpolation formula. (3)

OR

(3)

If $u = \left(\frac{x+y}{\sqrt{x+\sqrt{y}}} \right)$ then test that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + \frac{1}{2} \cot u = 0$.

Group-C
(Long Answer Type Questions)

5 x 6=30

7. Evaluate the characteristic equation of the matrix $A = \begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$. Justify that the characteristic equation satisfied by the matrix A. (5)

8. Explain the advantages and disadvantages of Newton-Raphson Method. (5)

9. Compute solutions of the system of equations by Matrix inversion method. (5)

$$x + 3y + 2z = 17$$

$$x + 2y + 3z = 16$$

$$2x - y + 4z = 13.$$

10. Let $v_1 = (1, -1, 0)$, $v_2 = (0, 1, -1)$ and $v_3 = (0, 0, 1)$ be elements of R^3 . Justify that the set of vectors $\{v_1, v_2, v_3\}$ is linearly independent. (5)

11. Write $f(0.23)$ and $f(0.29)$ using suitable interpolation formula from the following table: (5)

| | | | | | | |
|------|--------|--------|--------|--------|--------|--------|
| x | 0.20 | 0.22 | 0.24 | 0.26 | 0.28 | 0.30 |
| f(x) | 1.6596 | 1.6698 | 1.6804 | 1.6912 | 1.7024 | 1.7139 |

12. $\frac{dy}{dx} = 2x + y, y(0) = 1, h = 0.1$ then using Euler's formula write the value of $y(0.4)$. (5)

OR

Justify that $f(4) = f(3) + \Delta f(2) + \Delta^2 f(1) + \Delta^3 f(1)$. (5)
