BRAINWARE UNIVERSITY

Term End Examination 2022
Programme – B.Tech.(CSE)-AIML-2021/B.Tech.(CSE)-DS-2021/B.Tech.(CSE)-AIML-2022/B.Tech.(CSE)-DS-2022
Course Name – Calculus & Linear Algebra
Course Code - BSCM102/BSCD102
(Semester I)

Full Marks : 60 **Time : 2:30 Hours**

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group-A
(Multiple Choice Type Question) 1 x 15=15

1. *Choose the correct alternative from the following :*

(i) Evaluate the improper integral $\int_{-\infty}^{\infty} \frac{1}{1+x^2} dx =$

a) π
b) 1
c) $\frac{\pi}{2}$
d) None of these

(ii) If $f(x)$ satisfy all the conditions of Rolle's theorem in $[a, b]$, then $f'(x)$ becomes zero__________ (Select the correct option)

a) only at one point in (a, b)
b) at two points in (a, b)
c) at least one point in (a, b)
d) none of these

(iii) Select the value of $\Gamma\left(\frac{1}{3}\right)\Gamma\left(\frac{2}{3}\right)$ is

a) $\frac{2\pi}{\sqrt{3}}$
b) $\frac{3\pi}{\sqrt{2}}$
c) $\frac{\pi}{\sqrt{3}}$
d) $\frac{\pi}{\sqrt{2}}$

(iv) Test the convergence of the sequence $\left\{1, \frac{1}{3}, \frac{1}{3^2}, \ldots\ldots, \frac{1}{3^n}, \ldots. \infty\right\}$

a) convergent
b) divergent
c) oscillatory
d) none of these

(v) Which of the following theorem can be applied to the function $f(x) = x^3$ in the interval $[1,3]$?

a) Rolle's Theorem
b) Lagrange's Mean Value Theorem
c) Cauchy's Mean Value Theorem
d) None of these

(vi) Compute $\int_0^{\infty} e^{-x^2} dx =$

a) π
b) $\sqrt{\pi}$
c) $\frac{\sqrt{\pi}}{2}$
d) $\frac{\pi}{2}$

(vii)

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For $x > 0$, Identify $\lim_{x \to 0} \frac{\sin x}{\sqrt{x}} =$

a) <p style="text-align: left;"> 1
b) <p style="text-align: left;"> 0
c) 2
d) None of these

(viii) $\lim_{n \to \infty} \left[\frac{n}{n^2} + \frac{n}{(n^2+1^2)} + \frac{n}{(n^2+2^2)} + \dots \right]$ is equal to

a) $-\pi/4$
b) 0
c) $\pi/4$
d) $\pi/3$

(ix) Select the Lagrange's form of remainder in Taylor's theorem

a) $\frac{h^n (1-\theta)^{(n-1)}}{(n-1)!} f^n(a+\theta h)$
b) $\frac{h^n (1-\theta)^{(n-p)}}{p(n-1)!} f^n(a+\theta h)$
c) $\frac{h^n}{n!} f^n(a+\theta h)$
d) None of these

(x) Estimate the sum of series $1 + 1/2 + 1/2^2 + \dots$ is

a) 2
b) $\frac{3}{2}$
c) $\frac{4}{3}$
d) $\frac{10}{9}$

(xi) Test the convergence of the sequence $\{x_n\}$, where $x_n = (-1)^{n-1}$, is a

a) Convergent sequence
b) Divergent sequence
c) Oscillating sequence
d) None of these

(xii) Evaluate $\lim_{n \to \infty} \frac{2n+1}{n+1} =$

a) 1
b) 2
c) 3
d) None of these

(xiii) Compute $\int_0^1 \frac{1}{\sqrt{1-x^2}} dx =$

a) π
b) 1
c) $\frac{\pi}{2}$
d) None of these

(xiv) If $f(x,y) = 0$, then evaluate $\frac{dy}{dx} =$

a) $\frac{f_x}{f_y}$
b) $\frac{f_y}{f_x}$
c) $-\frac{f_x}{f_y}$
d) $-\frac{f_y}{f_x}$

(xv) If $u = \log \frac{x^2}{y}$ then evaluate $xu_x + yu_y =$

a) u
b) 0
c) 1
d) 2u

Group-B
(Short Answer Type Questions)

3 x 5=15

2. Show that $\frac{(b-a)}{\sqrt{1-a^2}} < \sin^{-1} b - \sin^{-1} a < \frac{(b-a)}{\sqrt{1-b^2}}$ if $0 < a < b < 1$ (3)

OR

Show that $\frac{(b-a)}{\sqrt{1-a^2}} < \sin^{-1} b - \sin^{-1} a < \frac{(b-a)}{\sqrt{1-b^2}}$ if $0 < a < b < 1$ (3)

3. Illustrate that $\frac{x}{1+x} < \log(1+x) < x$ for all $x > 0$. (3)

OR

Use mean value theorem illustrate $0 < \frac{1}{x} \log \frac{e^x - 1}{x} < 1$, for $x > 0$. (3)

4. Apply Maclaurin's theorem to the function $f(x) = (1+x)^4$ to deduce that $(1+x)^4 = 1 + 4x + 6x^2 + 4x^3 + x^4$. (3)

OR

Determine $\int_0^\infty e^{-x^4} x^2 dx \times \int_0^\infty e^{-x^4} dx = \frac{\pi}{8\sqrt{2}}$. (3)

5. If $u_n = \frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \cdots + \frac{1}{n(n+1)}$, then explain that the sequence $\{u_n\}$ is monotonically increasing and bounded. (3)

OR

If $u = \cos^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$ then conclude that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + \frac{1}{2}\cot u = 0$ (3)

6. Test the extrema of the following function: (3)

$$f(x,y) = x^3 + 3xy^2 - 3y^2 - 3x^2 + 4$$

OR

If $u = \log r$ and $r^2 = x^2 + y^2 + z^2$, Express that $r^2\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) = 1$. (3)

Group-C

(Long Answer Type Questions)

5 x 6=30

7. Conclude that the series $\frac{1}{1^p}+\frac{1}{2^p}+\frac{1}{3^p}+\ldots\ldots$ converges for $p>1$ and diverges for $p\leq 1$. (5)

8. Tell whether the vectors (1,1,0), (1,0,1) and (0,1,1) form a basis of R^3 over the set of real numbers. (5)

OR

Identify the linear mapping $T:R^3\rightarrow R^3$ which maps the basis vectors (0,1,1), (1,0,1), (1,1,0) of R^3 to (1,1,1), (1,1,1), (1,1,1) respectively. (5)

9. Use the Gram-Schmidt process of orthonormalisation to identify the orthonormal basis for the sub-space of R^4 generated by the vectors (1,1,0,1), (1, -2,0,0), (1,0,-1,2). (5)

OR

Identify the values of a,b,c if the matrix $A=\begin{bmatrix}0 & 2b & c\\ a & b & -c\\ a & -b & c\end{bmatrix}$ is orthogonal. (5)

10. Solve $\iint_A (x^2+y^2)\,\mathrm{dxdy}$; where A is the area between the line $y=x$ and $y=-x$ from $x=0$ to $x=1$. Verify that the change of order does not alter the value of the integration. (5)

OR

Determine whether the limit of $f(x,y)=\frac{xy}{y^2-x^2}$ exist when $(x,y)\rightarrow(0,0)$. (5)

11. If $\begin{vmatrix}x^3+3x & x-1 & x+3\\ x+1 & 1-2x & x-4\\ x-2 & x+4 & 3x\end{vmatrix}=ax^4+bx^3+cx^2+dx+e$ be an identity in x where a,b,c,d,e are constants, then calculate the value of e. (5)

OR

Conclude that every square matrix can be uniquely expressed as sum of symmetric and skew symmetric matrix. (5)

12. Evaluate $\iiint (x+y+z+1)^4\,dxdydz$ over the region bounded by $x\geq 0,\ y\geq 0,\ z\geq 0,\ x+y+z\leq 1$. (5)

OR

(5)

Justify $\int_0^1 \frac{1}{\sqrt{(1-x^4)}}dx = \frac{\sqrt{\pi}}{4} \times \frac{\Gamma\left(\frac{1}{4}\right)}{\Gamma\left(\frac{3}{4}\right)}$

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