Term End Examination 2022
Programme – M.Sc.(MATH)-2022
Course Name – Ordinary Differential Equations
Course Code - MSCMC104
(Semester I)

Full Marks : 60 **Time : 2:30 Hours**

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group-A
(Multiple Choice Type Question) 1 x 15=15

1. *Choose the correct alternative from the following :*

(i) Select the correct option. Picard's iteration formula is

a) $y^{(n+1)}(x) = y_0 + \int_{x_0}^{x} f(x, y^{(n)}(x))\,dx$

b) $y^{(n+1)}(x) = y_0 + \int_{x_0}^{x} f(x, y^{(n-1)}(x))\,dx$

c) $y^{(n+1)}(x) = y_0 + \int_{x_0}^{x} f(x, y^{(n+1)}(x))\,dx$

d) None of these.

(ii) Select the correct option. The general form of a linear second order differential equation is

a) $y'' + a_1(x)y' + a_0(x)y = \varphi(x)$

b) $y' + a_0(x)y = \varphi(x)$

c) $a_0(x)y = 0$

d) None of these.

(iii) Select the correct option. If x=0 is a regular singular point of the equation $y'' + P(x)y' + Q(x)y = 0$,then solution can be taken of the form

a) $y = \sum_{n=0}^{\infty} a_n x^{m+n}$

b) $y = \sum_{n=0}^{\infty} a_n x^n$

c) $y = \sum_{n=0}^{\infty} a_n x^{m+n}$

d) None of these.

(iv) Determine the order of the differential equation $\frac{d^3y}{dx^3} - 5x\frac{dy}{dx} = e^x + 1$

a) Third order
b) Second order
c) First order
d) None of these.

(v) Determine the unknown function and independent variable of $\frac{d^3y}{dx^3} - 5x\frac{dy}{dx} = e^x + 1$

Determine the unknown function and independent variable of $\frac{d^3y}{dx^3} - 5x\frac{dy}{dx} = e^x + 1$

y, e^x

Determine the unknown function and independent variable of

$\ddot{t}y + t^2 y - (\sin t)\sqrt{y} = t^2 - t + 1$

$2xy' + x^2 y' - (\sin x)\,y = 2$

$3y' + xy = e^{-x^2}$

$y'' - y = 0$

Analysis of boundary value problem involves functions of a differential operator. These functions are

a) algebraic function

b) Eigen function

c) logical function

d) symmetric function

(ix)

Justify if the differential equation $\frac{d^2y}{dx^2} + 16y = 0,$ for y(x) with the two boundary conditions $y_2(0) = 1,$ and $y_1\left(\frac{\pi}{2}\right) = -1$ then it has

a) no solution

b) exactly two solutions

c) exactly one solution

d) infinitely many solutions

(x) Select the correct option. Which of the following differential equation has constant co-efficients?

a) $y'' - y = 0$

b) $2xy' + x^2 y' - (\sin x)\,y = 2$

c) $3y' + xy = e^{-x^2}$

d) None of these.

(xi) Select the correct option. Which of the following differential equation is non-linear?

a) $yy''' + xy' + y = x^2$

b) $y'' - y = 0$

c) $3y' + xy = e^{-x^2}$

d) None of these

(xii)

Solve the equation $x\frac{dy}{dx} + y = 0,$ given that the solution graph is passing through the point (1,1) is:

a) <p style="text-align: left;"> x

b) x^2

c) x^{-1}

d) x^{-2}

(xiii)

Express the characteristic curves of the PDE $\left(1 - x^2\right)u_{xx} - u_{yy} = 0$ in the hyperbola case are

a) rectangular hyperbola

b) $\xi = y + \sin x, \eta = y - \sin x$

$\xi = y - \sin^{-1} x, \eta = y + \sin^{-1} x$

$\xi = y + \sin x, \eta = y - \sin x$

$\xi = y + \cos x, \eta = y - \cos x$

Select the correct option. $y(x) = 2e^{-x} + xe^{-x}$ is a solution of

$y'' + 2y' + y = 0$

$y'' + 4y' + y = 0$

$y'' + 6y' + y = 0$

$y'' + 10y' + y = 0$

Select the correct option. Using the substitution $x = e^z$, the equation

$x^2 \frac{d^2 y}{dx^2} - 5y = \log x$, reduces to

$\frac{d^2 y}{dz^2} + \frac{dy}{dz} - 5y = z$

$\frac{d^2 y}{dz^2} - \frac{dy}{dz} - 5y = z$

$\frac{d^2 y}{dz^2} - 2\frac{dy}{dz} - 5y = z$

(Short Answer Type Questions) 3 x 5=15

2. Find the general solution of the differential equation (3)

$$(x^2 + 1)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 6(x^2 + 1)^2,$$

given that $y = x$ and $y = x^2 - 1$ are two linearly independent solutions of the corresponding homogeneous equation.

3. Solve $y'' - y' - 2y = 4x^2$ given $y(0) = 1, y'(0) = 4$. (3)

4. Prove the orthogonality of Hermite Polynomials. (3)

5. Find a second solution y_2 linearly independent to the solution $y_1(t) = t$, of the differential equation $t^2y'' + 2ty' - 2y = 0$. (3)

OR

Find every solution y of the ODE $3t^2 + 4y^3y' - 1 + y' = 0$, leave the solution in implicit form. (3)

6. (3)

Let f be a real function defined on a domain D of the xy plane. State a sufficient condition for the unction f to satisfy a Lipschitz condition in D. Is the condition also necessary? Justify your answer with an example.

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Let f be a real function defined on a domain D of the xy-plane. State a sufficient condition for the unction f to satisfy a Lipschitz condition in D. Is the condition also necessary? Justify your answer with an example.

Develop the method of variation of parameters for solving the general second order linear differential equation with variable coefficients. (3)

Group-C

(Long Answer Type Questions)

5 x 6=30

7. Define a bounded function with suitable example and write the difference between a domain and a closed domain. (5)

8. Prove that $\frac{d}{dx}\left\{x^{-n}J_n(x)\right\} = -x^{-n}J_{n+1}(x)$. (5)

Find the value of $J_{\frac{-1}{2}}(x)$

9. Use the Sturm Separation Theorem to show that between any two consecutive zeros of $sin2t + cos2t$, there is precisely one zero of $sin2t - cos2t$. (5)

OR

Write down the formula for $J_n(x)$, then evaluate the value of $J_{\left(\frac{1}{2}\right)}(x)$ and $J_{\left(-\frac{1}{2}\right)}(x)$. (5)

10. Find the solution of the Laguerre's equation $x\frac{d^2y}{dx^2}+(1-x)\frac{dy}{dx}+ny=0$ (5)

11. Find all solutions of the Legendre equation $(1-x^2)y''-2xy'+l(l+1)y=0$, where l is any real constant. using power series centered at x=0. (5)

OR (5)

Determine the characteristic values and characteristic functions of the following Sturm-Liouville problem:

$$\frac{d^2y}{dx^2} + \lambda y = 0, \ y(1) = 0, \ y(L) = 0, \quad \text{where } L > 0.$$

(5)

Prove that the solution of the non-homogeneous equation $L[y] = f$ with homogeneous boundary conditions $U_1[y] = 0$, $U_2[y] = 0$, is given by

$$v(x) = \int_a^b G(x, \xi) f(\xi) d\xi$$

where $G(x, \xi)$ is a Green's function, given that the corresponding homogeneous boundary value problem has only trivial solution.

OR

If a nonzero function y_1 is solution to $y'' + a_1(t)y' + a_0(t)y = 0$, (5)

where a_1, a_0 are given functions then a second solution not proportional to y_1 is

$$y_2(t) = y_1(t) \int \frac{e^{-A_1(t)}}{y_1^2(t)} dt,$$

where $A_1(t) = \int a_1(t) dt$.

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