BRAINWARE UNIVERSITY

Term End Examination 2025-2026
Programme – M.Sc.(MATH)-2024
Course Name – Applied Numerical Analysis
Course Code - MSCMC402
(Semester IV)

Full Marks : 60 **Time : 2:30 Hours**

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group-A

(Multiple Choice Type Question) 1 x 15=15

1. *Choose the correct alternative from the following :*

(i) Identify the correct option. In Jacobi's method the elements of the orthogonal matrix S_1 are defined as

a) $s_{ij} = -\sin\theta, s_{ji} = \sin\theta, s_{ii} = \sin\theta, s_{jj} = \cos\theta$

b) $s_{ij} = -\sin\theta, s_{ji} = \sin\theta, s_{ii} = \cos\theta, s_{jj} = \cos\theta$

c) $s_{ij} = -\sin\theta, s_{ji} = \sin\theta, s_{ii} = \cos\theta, s_{jj} = -\cos\theta$

d) None of these

(ii) Select the correct option. The number of linearly independent eigenvectors of $\begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$ is

a) 0

b) 1

c) 2

d) 3

(iii) Choose the correct option. $\frac{dy}{dx} = f(x,y), y(x_0) = y_0$, then

$y_{n+1} = y_n + \frac{h}{2}[f(x_n, y_n) + f(x_{n+1}, y_n + hf(x_n, y_n))]$ is the formula of

a) Taylor's series method

b) Euler's method

c) Euler's modified method

d) Runge-Kutta method

(iv) **Choose the correct option. Milne's corrector formula is**

a) $y_{n+1} = y_n + \frac{h}{3}(y'_{n-1} + 4y'_n + 4y'_{n+1})$

b) $y_{n+1} = y_{n-1} + \frac{h}{3}(y'_{n-1} + 4y'_n + y'_{n+1})$

c) $y_{n+1} = y_n + \frac{4h}{3}(y'_{n-1} + 4y'_n + 4y'_{n+1})$

d) None of these

(v) Given $3\frac{dy}{dx} + 5y^2 = \sin x, y(0.3) = 5$ and step length $h = 0.3$. The value of $y(0.9)$ using Runge-Kutta 4th order method is mostly nearly_____.

Identify the correct answer.

a) -0.25011×10^{40}

b) -4297.4

c) -1261.5

d) 0.88498

(vi) Separate the correct answer. To find the values at the current time-step, the Crank-Nicolson scheme uses _____

a) t-Δ t and t-2Δ t steps

b) t-Δt and t+Δ t steps

c) t+Δ t and t+2Δ t steps

d) t and t+Δt steps

(vii) Select the correct option. According to the Adams-Moulton scheme, the derivative of a function T at time-step t is given by _____

a) $(3T(t)+4T(t-\Delta t)-T(t-2\Delta t))/2\Delta t$

b) $(3T(t)-4T(t-\Delta t)-T(t-2\Delta t))/2\Delta t$

c) $(3T(t)+4T(t-\Delta t)+T(t-2\Delta t))/2\Delta t$

d) $(3T(t)-4T(t-\Delta t)+T(t-2\Delta t))/2\Delta t$

(viii) Select the correct option. The Adams-Moulton scheme comes under _____

a) Backward schemes

b) Forward schemes

c) Multipoint schemes

d) Runge-Kutta methods

(ix) Select the correct option. If a method is stable for some values of the parameters then the method is called

a) Stable

b) Conditionally stable

c) unstable

d) None of these

(x) The speed of propagation of the disturbance in hyperbolic problems is _____. Select the correct option.

a) Finite

b) infinite

c) 0

d) 1

(xi) Judge the correct option. The two-dimensional heat flow equation is

a) $\frac{\partial u}{\partial t} = \alpha\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right]$

b) $\frac{\partial u}{\partial t} = \alpha^2\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right]$

c) $\frac{\partial u}{\partial t} = \alpha^2\left[\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2}\right]$

d) $\frac{\partial u}{\partial t} = \alpha\left[\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2}\right]$

(xii) Choose the correct digit that is not significant in the number 0.000564.

a) 5

b) 4

c) 6

d) 0

(xiii) Choose the hyperbolic equation.

a) Unsteady Navier-Stokes equation

b) Steady Navier-Stokes equation

c) Steady Euler equation

d) Unsteady Euler equation

(xiv) Choose the correct option. The eigen vector for the eigenvalue $\lambda = 3$ of the given matrix $A = \begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$ is____.

a) $\begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix}$

b) $\begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$

c) $\begin{bmatrix} -1 \\ -1 \\ -2 \end{bmatrix}$

d) $\begin{bmatrix} -1 \\ -2 \\ 2 \end{bmatrix}$

(xv) Choose the correct point that gives the value from the first two steps of the fourth-order Runge-Kutta method.

a) At the $(n+0.5)^{th}$ point

b) At the $(n+1)^{th}$ point

c) At the $(n-1)^{th}$ point

d) At the n^{th} point

Group-B

(Short Answer Type Questions) 3 x 5=15

2. Describe the statement of Cayley-Hamilton theorem and justify the theorem for the matrix $\begin{bmatrix} 6 & 4 \\ 1 & 3 \end{bmatrix}$. (3)

3. Estimate the largest eigenvalue and eigenvector of the given matrix $\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$ by applying Power method. (3)

4. Explain predictor-corrector method for solving ordinary differential equation. (3)

5. Given $\frac{dy}{dx} = x^2(1+y)$ and $y(1) = 1, y(1.1) = 1.233, y(1.2) = 1.548, y(1.3) = 1.979, y_4^p = 2.573$, where y_4^p is the predictor value of y at $x = 1.4$. Evaluate $y(1.4)$ by Adams-Bashforth method. (3)

6. Explain the Standard Five Point formula for the Laplacian equation. (3)

OR

Explain the error in solving Laplace and Poisson's equations by finite difference method. (3)

Group-C

(Long Answer Type Questions) 5 x 6=30

7. Given $A = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$. Estimate the eigenvalues and corresponding eigenvectors of the given matrix using Power method. (5)

8. Evaluate the solution of the equation $\nabla^2 u = -10(x^2 + y^2 + 10)$ over the square with sides $x = 0 = y, x = 3 = y$ with $u = 0$ on the boundary and mesh length =1. (5)

9. Deduce predictor formula Adams-Bashforth predictor-corrector formula. (5)

10. Write the advantages and disadvantages of the Jacobi's method. (5)

11. Explain the classification of the general second order PDE and write the general form of all type of equations. (5)

12. Evaluate the solution of the equation $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$ with the boundary conditions $u = 0$ at $x = 0$ and $1, t > 0$ and the initial conditions $u = \frac{1}{2}\sin \pi x, \frac{\partial u}{\partial t} = 0$, when $t = 0, 0 \leq x \leq 1$, for x=0.2 and $t = 0.1$. (5)

OR

Evaluate the value of $y(0.20)$ from the differential equation $y' = 2y - y^2$ with $y(0) = 1$ taking step size $h = 0.05$ by applying Adams-Bashforth-Moulton predictor-corrector method. (5)

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