BRAINWARE UNIVERSITY

Term End Examination 2025-2026
Programme – M.Sc.(MATH)-2024
Course Name – Manifolds and Applications
Course Code - MSCME401A
(Semester IV)

Full Marks : 60 **Time : 2:30 Hours**

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group-A

(Multiple Choice Type Question) 1 x 15=15

1. *Choose the correct alternative from the following :*

(i) Identify the correct option. A topological manifold must be Hausdorff to ensure that _____

a) It is compact
b) Limits of sequences are unique
c) It has a metric
d) It is finite-dimensional

(ii) Identify the correct option. All the tangent vectors at a particular point in a manifold forms a/an _____

a) Tangent space
b) Chart
c) Atlas
d) Tangent bundle

(iii) Select the correct option. If dF is injective for all the points, the map F is called an _____

a) Submersion
b) Immersion
c) Diffeomorphism
d) Homeomorphism

(iv) Select the correct option. The Lie bracket [X,Y] of two vector fields is _____

a) XY + YX
b) XY - YX
c) X(Y)
d) None of these

(v) Recognize the correct option. A 1-form W is called closed if _____

a) dW = 0
b) W = 0
c) It is defined on a compact manifold (d) It has no boundary
d) It has no boundary

(vi) Identify the correct option. A curve is a geodesic if its velocity vector is _____

a) Constant in length
b) Parallel transported along itself
c) Perpendicular to the metric
d) Vanishing at infinity

(vii) Identify the correct option. In the context of differential geometry, a hypersurface is defined as a submanifold of _____

a) Dimension reduced by 1 b) Codimension 2
c) Infinite dimension d) Dimension 0

(viii) Identify the correct option. _____ is the Gaussian curvature K of a sphere of radius a.

a) a b) 1/a
c) Square of 1/a d) 0

(ix) Choose the correct option. Every orientable 2-dimensional manifold is _____

a) Almost complex but not complex. b) Never almost complex.
c) Always a complex manifold. d) Only complex if it is 2-dimensional sphere

(x) Choose the correct option. On an almost complex manifold, the tangent bundle TM itself admits _____

a) No structure b) An almost complex structure canonically
c) Only a metric structure d) A unique complex structure

(xi) Select the correct option. A one-parameter group of transformations on a manifold is generated by _____

a) A scalar field b) A vector field
c) A tensor field d) A connection

(xii) Select the correct option. In a coordinate chart, the tangent vectors are represented by _____

a) Partial derivatives b) Total derivatives
c) Covariant derivatives d) Lie derivatives

(xiii) Select the correct option. The Riemann curvature tensor R(X, Y)Z measures _____

a) The failure of parallel transport to commute b) Torsion
c) Scalar curvature d) Geodesic deviation

(xiv) Select the correct option. In Riemannian geometry, the curvature tensors satisfy _____

a) Bianchi identities b) Jacobi identities
c) Both A and B d) None of these

(xv) Distinguish the correct option. A Riemannian manifold is a smooth manifold with _____

a) A complex structure b) A symplectic form
c) A positive-definite metric tensor d) A torsion-free connection

Group-B

(Short Answer Type Questions) 3 x 5=15

2. Illustrate the Koszul's formula for a Riemannian manifold. (3)

3. State the difference between a C^1 and a C^∞ (smooth) manifold. (3)

4. Define the tangent space T_pM using the derivation approach. (3)

5. Describe a hypersurface and provide its codimension. (3)

6. Explain integral curve for a vector field. (3)

OR

Illustrate the formula of geodesic equation. (3)

Group-C

(Long Answer Type Questions) 5 x 6=30

7. Show that (5)

$$[X, [Y, Z]] + [Y, [Z, X]] + [Z, [X, Y]] = 0 .$$

8. Determine the integral curves of the following vector field on $\mathbb{R}^2$: (5)

$$x^2 \frac{\partial}{\partial x^1} - x^1 \frac{\partial}{\partial x^2}$$

9. In a Riemannian manifold (M, g), deduce that (5)

$$\Gamma_{ijk} + \Gamma_{kji} = \frac{\partial g_{ik}}{\partial u^j} .$$

10. If R is the Riemann curvature in a Riemannian manifold (M, g), deduce that (5)

$$R(X, Y)Z + R(Y, Z)X + R(Z, X)Y = 0 .$$

11. If ∇ is the Levi-Civita connection and R is the Riemann curvature, deduce that (5)

$$(\nabla_X R)(Y, Z) + (\nabla_Y R)(Z, X) + (\nabla_Z R)(X, Y) = 0 .$$

12. If K is a tensor field of type (0,1) on a Riemannian manifold (M, g), then deduce that (5)

$$(\nabla_X \nabla_Y K)(Z) - (\nabla_Y \nabla_X K)(Z) - (\nabla_{[X,Y]} K)Z = -K(R(X, Y)Z) .$$

OR

Deduce that a real vector space with a complex structure F is even dimensional. (5)

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