BRAINWARE UNIVERSITY

Term End Examination 2022
Programme – M.Sc.(MATH)-2019/M.Sc.(MATH)-2021
Course Name – Functional Analysis
Course Code - MSCMC301
(Semester III)

Full Marks : 60 Time : 2:30 Hours

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group-A
(Multiple Choice Type Question) 1 x 15=15

1. *Choose the correct alternative from the following :*

(i) Select which of the following is not norm linear space over Unitary Space?

a) The set of all convergent sequences in Unitary space.
b) The set of all bounded sequences in Unitary space.
c) The set of all sequences in Unitary space that converges to 0.
d) The set of all sequences in Unitary space that converges to a real number

(ii) Let $\|\|$ be a norm on a vector space X and $\|x+y\|=0$. State the true statement.

a) $x=y=0$
b) $x=-y$
c) $\|x\|=\|y\|$
d) $\|x\|=-\|y\|$

(iii) Let $\|\|$ be a norm on a vector space X. State the true statement.

a) $\|y\|-\|x\|\le\|y-x\|$
b) $\|x\|-\|y\|\le\|y-x\|$
c) $|\,\|y\|-\|x\|\,|\le\|y-x\|$
d) $\|y\|-\|x\|=\|y-x\|$

(iv) Identify the wrong statement in a Banach space.

a) Every Cauchy sequence is convergent
b) Every convergent sequence is Cauchy
c) Every sequence has a convergent subsequence
d) Every sequence is convergent

(v) Select the correct statement for a bounded linear operator $T: X\to Y$.

a) $\|Tx\|=\|x\|$
b) $\|Tx\|\le\|x\|$
c) $\|Tx\|\ge c\|x\|$
d) $\|Tx\|\le c\|x\|$

(vi)

Let $I : X \to X$ be the identity linear transformation, where X is a normed space and $X \neq \{0\}$ and 0 is the zero vector of X. Compute $\|I\|$.

a) -1
b) 1
c) 1 or -1
d) None of these

(vii) Let T be an unbounded linear operator on a normed space X. Identify the <div>correct statement</div>

a) X is finite dimensional
b) X is infinite dimensional
c) T(X) is finite dimensional
d) None of the mentioned

(viii) In a metric space X if a subset M of X is compact then write about M.

a) M is only closed
b) M is only bounded
c) M is both closed and bounded
d) M may not be closed or bounded

(ix) Test the correctness of the following statement.

a) A subset M of a metric space X is compact if M is closed
b) A subset M of a metric space X is compact if M is bounded
c) A subset M of a metric space X is compact if M is both closed and bounded
d) None of these

(x) If in a normed space X has the property that the closed unit ball is compact <div>then justify the statement.</div>

a) X is finite dimensional
b) X is infinite dimensional
c) X is compact
d) None of these

(xi) Let $T : X \to Y$ be a bounded linear operator between normed spaces X and Y. Then test the topological property of the null space of T.

a) closed
b) open
c) both closed and open
d) None of these

(xii) Choose the correct statement for a bounded, linear and one-one operator T from a Banach space X onto a Banach space Y with the property that for every sequence $\{x_n\} \to 0$ in X implies $\{Tx_n\} \to 0$ in Y.

a) T is continuous on X
b) T is only continuous at 0 in X
c) T is continuous in X except at 0
d) None of these

(xiii) Select what Norm convergence is known as :

a) Uniform convergence
b) Strong convergence
[redacted]
c) Weak convergence.
d) None of these.

(xiv) Select the true property for any sublinear functional p

a) $| p(x) - p(y) | \leq p(x - y)$
b) $| p(x) - p(y) | \geq p(x - y)$
c) $| p(x) - p(y) | = p(x - y)$
d) None of these

(xv) Let Y be a closed subspace of a Hilbert space X. Then Y is concluded to be

a) compact
b) complete
c) convex but not complete
d) convex but not compact

Group-B
(Short Answer Type Questions) 3 x 5=15

2. (3)

Compute all the rare sets in a discrete metric space X.

3. Show that every convergent sequence in a metric space is a Cauchy sequence (3)

OR

Show that $d(x,y)=\|x-y\|, x,y \in X$ is a metric on the NLS $(X,\|.\|)$ (3)

4. Determine the span of $M=\{(1,1,1),(0,0,2)\}$ in R^3. (3)

OR

Write an example of an NLS which has a Schauder basis. (3)

5. Explain the concept of Schauder basis on a normed linear space (NLS). (3)

OR

Let T be a bounded linear operator. Then conclude that the null space of T is closed. (3)

6. Justify the statement, "If a normed space X is finite dimensional, then every linear operator on X is bounded." (3)

OR

Write an example of bounded linear operator whose inverse is unbounded operator. (3)

Group-C

(Long Answer Type Questions) 5 x 6=30

7. Let T be a bounded linear operator. Then deduce that $x_n \to x \Rightarrow Tx_n \to Tx$ (5)

8. Define Hilbert spaces and Banach Spaces (5)

OR

Define the resolvent set and spectrum of a liner operator. (5)

9. Show that every finite dimensional subspace of a normed linear space is complete. (5)

OR

(5)

Show that similar matrices have same eigenvalues.

10. Explain Baire's Category theorem and Hanh Banach theorem (5)

OR

Explain the open mapping theorem and closed graph theorem. (5)

11. Justify the statement, "If a normed space X has a Schauder basis, then X is separable." (5)

OR

Justify that any finite dimensional linear subspace of norm linear space is closed. (5)

12. Let X be a finite dimensional vector space. If $x_1 \in X$ has the property that $f(x_1) = 0, \forall f \in X'$ then deduce that $x_1 = 0$. (5)

OR

If X is a finite dimensional vector space, then deduce that any two norms on X are equivalent. (5)

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