BRAINWARE UNIVERSITY

Term End Examination 2022
Programme – M.Sc.(MATH)-2019/M.Sc.(MATH)-2021
Course Name – Differential Geometry
Course Code - MSCMC302
(Semester III)

Full Marks : 60   Time : 2:30 Hours

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group-A
(Multiple Choice Type Question)   1 x 15=15

1. *Choose the correct alternative from the following :*

(i) A^i and B^i are two vectors of constant magnitudes and undergo parallel displacements along a given curve, then evalua the angle they are inclined is

a) 0
b) $\frac{\pi}{2}$
c) Constant
d) None of these

(ii) A space curve is called a helix if the tangent at any point of it makes an angle with a fixed direction. Identify the angle.

a) 0
b) $\frac{\pi}{2}$
c) Constant
d) None of these

(iii) Identify the rectangular Cartesian coordinates of the point whose cylindrical coordinates are $\left(2,\frac{\pi}{3},1\right)$ are

a) $\left(1,\sqrt{3},1\right)$
b) $(1,2,1)$
c) $(1,3,1)$
d) None of these

(iv) A twisted curve $(\tau \neq 0)$ is a Bertrand curve if and only if its curvature (κ) and torsion (τ) are expressed by a linear equation

a) $a\kappa + b\tau = 0$, a,b are non zero constants.
b) $a\kappa + b\tau = 1$, a,b are non zero constants.
c) $a\kappa = b\tau$, a,b are non zero constants.
d) None of these

(v) Express the first fundamental quadratic form of the surface

a) $a_{\alpha\beta} du^\alpha du^\beta$, where $a_{\alpha\beta} = \sum_i \frac{\partial x^i}{\partial u^\alpha}\frac{\partial x^i}{\partial u^\beta}$, $\alpha, \beta = 1, 2$
b) $\frac{1}{\tau}\left(\frac{\delta \mu^i}{\delta s} + \kappa\lambda^i\right)$
c) $g_{ij}\left(x^i - x_0^i\right)\nu^j$
d) $-\tau\mu^\kappa$

(vi) ψ is called invariant if $\frac{\partial \overline{\psi}}{\partial \overline{x}^i} =$ (Select the correct option)

a) $\frac{\partial \psi}{\partial x^j}\cdot\frac{\partial x^j}{\partial \overline{x}^i}$
b) $\frac{\partial \psi}{\partial x^i}\cdot\frac{\partial x^i}{\partial \overline{x}^j}$
c)
d)

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None of these

$\frac{\partial x^i}{\partial x^j}$

(vii) If A^i and B_i are a contravariant and a covariant vector respectively, then identify $A^i B_i$ is

a) <p style="text-align: left;"> 0
b) Invariant
c) <p style="text-align: left;"> 1
d) None of these

(viii) If all the components of a tensor in one coordinate system are zero at a point, then identify their value at this point in every coordinate system is

a) Not specific
b) <p style="text-align: left;"> 1
c) <p style="text-align: left;"> 0
d) None of these

(ix) Identify the type of tensor δ^i_j

a) $(1,1)$
b) $(1,0)$
c) $(1,2)$
d) None of these

(x) If two covariant indices of a tensor can be interchanged without altering the tensor, then it is said to be (Choose the correct Option)

a) Symmetric
b) Skew-symmetric
c) Constant
d) None of these

(xi) If two contravariant indices of a tensor can be interchanged without altering the tensor, then it is said to be (Choose the correct Option)

a) Symmetric
b) Skew-symmetric
c) Constant
d) None of these

(xii)

Determine the kronecker delta δ^i_j is

a) Symmetric in i and j
b) Skew-symmetric in i and j
c) 1
d) None of these

(xiii) The intrinsic derivative of a vector A along a curve C vanishes at all points of C, then choose magnitude of A along the curve C is

a) 0
b) Constant
c) Not specific
d) None of these

(xiv) A space curve is a straight line if and only if its curvature is (Select the correct option)

a) 0
b) 1
c) Any arbitrary constant
d) None of these

(xv) Serret – Frenet formulae for a space curve are (Select the correct option)

a) $\mu^i = \frac{1}{\kappa}\frac{\delta\lambda^i}{\delta s}$ and $\nu^i = \frac{1}{\tau}\left(\frac{\delta\mu^i}{\delta s} + \kappa\lambda^i\right)$

b) $\mu^i = \frac{1}{\kappa}\frac{\delta\lambda^i}{\delta s}$ and $\frac{\delta\nu^k}{\delta s} = -\tau\mu^k$

c) $\mu^i = \frac{1}{\kappa}\frac{\delta\lambda^i}{\delta s}$, $\nu^i = \frac{1}{\tau}\left(\frac{\delta\mu^i}{\delta s} + \kappa\lambda^i\right)$ and $\frac{\delta\nu^k}{\delta s} = -\tau\mu^k$

d) None of these

Group-B
(Short Answer Type Questions) 3 x 5=15

2. Illustrate Christoffel symbols vanish identically if and only if g_{ij}'s are constants. (3)

3. Show that Christoffel symbol of first kind is symmetric in first two indices. (3)

OR

Christoffel symbol of second kind is symmetric in second two indices. (3)

4. (3)

Illustrate the fundamental metric tensor g_{ij} is symmetric.

OR

Determine kronecker delta δ^i_j is a mixed tensor of order two. (3)

5. (3)

Explain $[ij,h] = g_{kh}\begin{Bmatrix} k \\ ij \end{Bmatrix}$

OR

Show that the contraction of a tensor of order $(2,3)$ is a tensor of order $(1,2)$. (3)

6. Express $\frac{\partial g^{im}}{\partial x^l} = -g^{lj}\begin{Bmatrix} m \\ lj \end{Bmatrix} - g^{km}\begin{Bmatrix} i \\ kl \end{Bmatrix}$ (3)

OR

Express $\frac{\partial g_{ij}}{\partial x^k} = g_{lj}\begin{Bmatrix} l \\ ik \end{Bmatrix} + g_{il}\begin{Bmatrix} l \\ jk \end{Bmatrix}$ (3)

Group-C

(Long Answer Type Questions) 5 x 6=30

7. If $U^i = A^i + B^i$, where A^i and B^i are two orthogonal unit vectors, illustrate that the square of the length of the vector U^i is 2. (5)

8. Identify that $\gamma(t)$ is a unit-speed curve iff t is the arc-length of $\gamma(t)$ is measured from some point (5)

OR

Let, $\gamma(s)$ be a unit speed curve with curvature $k(s) > 0$ and torsion $\tau(s) \neq 0\ \forall\, s$. Then identify that if γ lies on the surface of a sphere, $\frac{\tau}{k} = \frac{d}{ds}\left(\frac{\frac{dk}{ds}}{\tau k^2}\right)$ (5)

9. If $\gamma(s)$ is a unit speed plane curve, then illustrate that $\frac{dn_s}{ds} = -k_s\,.\,t$ (5)

OR

A unit speed plane curve $\gamma(s)$ has the property that its tangent vector $t(s)$ makes a fixed angle θ with $\gamma(s)\ \forall\, s$. Illustrate that if $\theta = 0$, then γ is a part of straight line. (5)

10. Illustrate that the number of independent components of the fundamental metric tensor is at most $\frac{n(n+1)}{2}$. (5)

OR

If $a^{ij}u_iu_j = b^{ij}u_iu_j$ for an arbitrary covariant vector u_i, Illustrate that $a^{ij} + a^{ji} = b^{ij} + b^{ji}$ (5)

11. In V_4 with line element $ds^2 = -(dx^1)^2 - (dx^2)^2 - (dx^3)^2 + c^2(dx^4)^2$. Illustrate that the vector $(\sqrt{2}, 0, 0, \frac{\sqrt{3}}{c})$ is a unit vector. (5)

OR

(5)

A unit speed plane curve $\gamma(s)$ has the property that its tangent vector $t(s)$ makes a fixed angle θ with $\gamma(s)\ \forall\ s$. Illustrate that if $\theta = \frac{\pi}{2}$, then γ is a part of circle.

(5)

12. Let us consider a V_4 with the line element given by $ds^2 = -(dx^1)^2 - (dx^2)^2 - (dx^3)^2 + c^2(dx^4)^2$. Justify that the vector $(-1,0,0,\frac{1}{c})$ is a null vector in this space.

OR

If the equality $\lambda^i_j A^j = \sigma A^i$ holds for any contravariant vector A^i, where σ is a scalar, justify that $\lambda^i_j = \sigma\delta^i_j$ (5)

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