BRAINWARE UNIVERSITY

Term End Examination 2022
Programme – M.Sc.(MATH)-2019/M.Sc.(MATH)-2021
Course Name – Integral Equations & Calculus of Variations
Course Code - MSCMC303
(Semester III)

Full Marks : 60 **Time : 2:30 Hours**

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group-A
(Multiple Choice Type Question) 1 x 15=15

1. *Choose the correct alternative from the following :*

(i) In $g(x)y(x) = f(x) + \lambda \int_a K(x,t)y(t)dt$, the upper limit

a) always a fixed value
b) may be variable x
c) may be fixed constant
d) either (b) or (c)

(ii) The geodesics of a right circular cylinder of radius a

a) circle
b) circular helix
c) parabola
d) None of these

(iii) The function K(x.t) of $g(x)y(x) = f(x) + \lambda \int_a K(x,t)y(t)dt$ is called….. of the integral equation.

a) Kernel
b) Integral
c) integral constant
d) None of these

(iv) A curve C of a given length l which minimizes the curved surface area of solid generated by the revolution C about x-axes is a

a) Sphere
b) Catenary
c) Ellipse
d) None of these

(v) If g(x)=0, in $g(x)y(x) = f(x) + \lambda \int_a K(x,t)y(t)dt$, then the equation is called

a) linear integral equation of 1st kind
b) linear integral equation of 2nd kind
c) linear integral equation of 3rd kind
d) None of these.

(vi) Which of the following is linear Fredholm Integral equation?

a) $g(x)y(x) = f(x) + \lambda \int_a^b K(x,t)y(t)dt$

b) $g(x)y(x) = f(x) + \lambda \int_a^x K(x,t)y(t)dt$

c)

d) None of these.

$$y(x) = \int_a^b K(x,t)[y(t)]^2 dt$$

(vii) The kernel of the integral equation $u(x) = x + \int_0^{\frac{1}{2}} u(t)\,dt$ (Fredholm Integral equation of 2nd kind) is

a) 0
b) 3
c) 2
d) 1

(viii) The integral equation $y(x) = \int_0^x (x-t)y(t)\,dt - x\int_0^1 (1-t)y(t)\,dt$ is equivalent to

a) $y'' - y = 0, y(0) = 0, y(1) = 0$
b) $y'' - y = 0, y(0) = 0, y'(0) = 0$
c) $y'' + y = 0, y(0) = 0, y(1) = 0$
d) $y' - y = 0, y(0) = 0, y'(0) = 0$

(ix) The solution of the integral equation $u(x) = \frac{23}{6}x + \frac{1}{8}\int_0^1 xy\,u(y)\,dy$ is

a) 2x
b) 3x
c) 4x
d) x

(x) The shortest distance between the lines (x–3)/3 = (y –8)/–1 = (z–3)/1, (x + 3)/–3 = (y+7)/2 = (z–6)/4 is

a) <p style="text-align: left;"> 1
b) <p style="text-align: left;"> 3
c) 3√30
d) <p style="text-align: left;"> other

(xi) Evaluate the kernel of the following integral equation and choose the correct option.

$$g(s) = s + \int_0^1 su^2 g(u)\,du$$

a) su^2
b) u^2
c) <p style="text-align: left;"> u
d) <p style="text-align: left;"> Other

(xii) Evaluate the class of the following integral equation $y(x) = \lambda \int_0^{2\pi} \sin(x + t)\, y(t)\,dt$.

a) <p style="text-align: left;"> Linear Fredholm
b) <p style="text-align: left;"> Linear Volterra
c) Non-Linear Fredholm
d) Other

(xiii) Let y(x) be the solution of the integral solution $y(x) = x - \int_0^x xt^2 y(t)\,dt, x > 0$

Then the value of the function y(x) at $x = \sqrt{2}$ is equal to

a) $\frac{1}{2\sqrt{e}}$
b) $\frac{e}{2}$
c) $\frac{\sqrt{2}}{e^2}$
d) $\frac{\sqrt{2}}{e}$

(xiv) The extremal of $\int_0^2 (\frac{y'^2}{x})\,dx$, y(0)=0 , y(2)=1 is

a) ellipse
b) circle
c) straight line
d) <p style="text-align: left;"> parabola

(xv) Fredholm Integral equation of second kind is

a) $y(x) = f(x) + \lambda \int_a^x K(x,t)y(t)dt$
b) $y(x) = f(x) + \lambda \int_a^b K(x,t)y(t)dt$
c) $y(x) = \lambda \int_a^b K(x,t)y(t)dt$
d) None of these.

Group-B

(Short Answer Type Questions)

3 x 5=15

(3)

2. If $y''(x) = F(x)$, and y satisfies the initial conditions $y(0) = y_0$ and $y'(0) = y'_0$ then show that $y(x) = y_0 + x y'_0 + \int_0^x (x - t)F(t)dt$

OR

(3)

Show that the solution of the Volterra equation $y(x) = 1 - \int_0^x (x - t)y(t)dt$ satisfies the differential equation $y''(x) + y(x) = 0$ and the boundary conditions $y(0) = 1,\ y'(0) = 1.$

(3)

3. Find the extremal of the functional $\int_{x_1}^{x_2}\{a(x)y'^2 + 2b(x)yy' + c(x)y^2\}dx$ is a second order linear differential equation.

OR

(3)

Find the extremal of the functional $\int_1^3 (3x - y)y dx$ that satisfy the boundary conditions y (1) =1, y (3) = $\frac{9}{2}$

(3)

4. Obtain the Euler's equation for the extremals of the functional

$$\int_{x_1}^{x_2} \{y^2 - yy' + y'^2\}dx$$

OR

(3)

Find the Laplace transformation of the delayed unit impulse function $\delta(t-1)$.

(3)

5. Reduce the following boundary value problem into an integral equation:
$\frac{d^2y}{dx^2} + \lambda y = 0$ with $y(0) = 0,\ y(l) = 0$

OR

(3)

Transform $\frac{d^2y}{dx^2} + xy = 1,\ \ y(0) = 0,\ y(1) = 0$ into an integral equation.

(3)

6. Convert the boundary value problem $y''(x) + y(x) = x,\ y(0) = 0,\ y'(1) = 0$ into an integral equation.

OR

(3)

If $y(x)$ is continuous and satisfies $y(x) = \lambda\int_0^1 K(x,t)y(t)dt$, where

$K(x,t) = \begin{cases}(1-t)x, & 0 \le x \le t \\ (1-x)t, & t \le x \le 1\end{cases}$, then prove that $y(x)$ is also the solution of the boundary value problem $\frac{d^2y}{dx^2} + \lambda y = 0,\ \ y(0) = 0,\ y(1) = 0.$

Group-C 5 x 6=30

(Long Answer Type Questions)

7. Show by an example that an integral equation possesses no solution for $f(x)=x$, but that it possesses infinitely many solutions when $f(x)=1$. (5)

8. Show that $y(x) = \cos cos\, 2x$ is a solution of the integral equation (5)

$$y(x) = cosx + 3\int_0^{\pi} K(x,t)y(t)dt \text{ where } K(x,t) = \begin{cases} sinx\, cost, & 0\le x\le t \\ cosx\, sint, & t\le x\le \pi \end{cases}$$

9. Define and establish the Tautochrone problem. (5)

OR

Define and solve Abel's integral equation.. (5)

10. Find the eigen value and eigen functions of the integral equation (5)

$$y(x) = \lambda\int_0^{\pi} K(x,t)y(t)dt$$

where $K(x,t) = cosx\, sint,\ 0\le x\le t$

$= cost\, sint,\ t\le x\le \pi$

OR

Solve the homogeneous Fredholm integral equation of the second kind: (5)

$$y(x)=\lambda\int_0^{2\pi}\sin(x+t)\,y(t)\,dt$$

11. Convert $y'' - sinx\, y' + e^{x}y(x) = x$ with initial conditions $y(0) = 1, \dot{y}(0) = -1$ to a Volterra equation of second kind. (5)

OR

Convert $y'' - 3\,y'(x) + 2y(x) = 4sinx$ with initial conditions $y(0) = 1, \ddot{y}(0) = -2$ to a Volterra equation of second kind. (5)

(5)

12. Show that the integral equation

$$y(x)=f(x)+\frac{1}{\pi}\int_0^{2\pi}\sin(x+t)\,y(t)\,dt$$

possesses no solution for $f(x)=x$, but that it possesses infinitely many solutions when $f(x)=1$.

OR

(5)

Show that

$$e^{-x}\cos x=\frac{2}{\pi}\int_0^{\infty}\frac{(u^2+2)\cos ux}{u^2+4}\,du,\; x>0$$

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