



(Semester II)

Library
Brainware University
398, Ramkrishnapur Road, Barasat
Kolkata, West Bengal-700125

Time : 2:30 Hours

Group-A

(Multiple Choice Type Question)

$$1 \times 15 = 15$$

1. Choose the correct alternative from the following :
 - (i) Choose the correct option. Let (G, o) be a group and $a, b \in G$. If $O(a) = 3$ and $aoboa^{-1} = b^2$, then for $b \neq e$, the $O(b)$ is
 - a) 4
 - b) 6
 - c) 7
 - d) 8
 - (ii) Identify the correct option. In Klein's 4-group $\{e, a, b, c\}$, the order of each non-identity elements is
 - a) 1
 - b) 2
 - c) 3
 - d) 4
 - (iii) Select the correct option. The number of elements in an alternative group A_n is
 - a) $n!$
 - b) $\frac{n!}{4}$
 - c) 2^n
 - d) $\frac{n!}{2}$
 - (iv) If $G = \{1, -1, i, -i\}$ is a multiplicative group, then identify the order of $-i$
 - a) 2
 - b) 4
 - c) 1
 - d) 8
 - (v) Select the correct option. The number of elements in a symmetric group S_n is

3. Show that every group of order 45 has a normal subgroup of order 9. (3)
4. Show that the polynomial $x^3 + x + 1$ is irreducible over $\mathbb{Z}_5$. (3)
5. Illustrate the definition of algebraic extension. Give an example of it. (3)
6. Justify that $SL(n, \mathbb{R})$ is a normal subgroup of $GL(n, \mathbb{R})$. (3)

OR

Let G be a group and $Z(G)$ be the centre of G . Then justify that if $a \in Z(G)$ then $cl(a) = \{a\}$ and conversely. (3)

Group-C

(Long Answer Type Questions)

5 x 6=30

7. Let H be a subgroup of a group G and $[G:H] = 2$. Justify that for every $x \in G$, $x^2 \in H$. (5)
8. Define normaliser $N(H)$ of a subgroup H of a group G and show that $N(H)$ is a subgroup of G . (5)
9. Let H be a subgroup of a group G and $a \in G$. Justify that the subset $aHa^{-1} = \{aha^{-1} : h \in H\}$ is a subgroup of G . (5)
10. Let S and T be two ideals of a ring R . Examine that $S + T$ is an ideal of R , where $S + T = \{s + t : s \in S, t \in T\}$. (5)
11. Evaluate $[\mathbb{Q}(\sqrt{3}, \sqrt{7}) : \mathbb{Q}(\sqrt{3})]$ and $[\mathbb{Q}(\sqrt{3}) : \mathbb{Q}]$. (5)

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12. Let G be a group and H be a normal subgroup of G . Test whether $(g, h) \mapsto ghg^{-1}$ is a group action of G on H or not. (5)

OR

- Let G be a non-commutative group of order p^3 , p a prime. Evaluate $|Z(G)|$. (5)

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