



Term End Examination 2025-2026

Programme – M.Sc.(MATH)-2025

Course Name – Measure Theory and Integration

Course Code - MSM20407

(Semester II)

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Brainware University
398, Ramkrishnapur Road, Barasat
Kolkata, West Bengal-700125

Full Marks : 60

Time : 2:30 Hours

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group-A

(Multiple Choice Type Question)

 $1 \times 15 = 15$

1. Choose the correct alternative from the following :

- (i) Identify the family which is closed under countable unions and complements.
- a) Ring
b) Algebra
c) σ -algebra
d) Semiring
- (ii) Identify the smallest σ -algebra containing all open sets in $\mathbb{R}$.
- a) Power set
b) Lebesgue σ -algebra
c) Borel σ -algebra
d) Algebra of intervals
- (iii) Select the Lebesgue measure of $(0,1)$.
- a) 0
b) 1
c) 2
d) None of these
- (iv) Select the correct answer. A always measurable set is
- a) Vitali set
b) Finite set
c) Arbitrary set
d) None of these
- (v) Select the class of functions whose product is measurable.
- a) Continuous
b) Measurable
c) Integrable
d) None of these
- (vi) Identify the function approximated in Lusin's theorem.
- a) Continuous
b) Measurable
c) Polynomial
d) None of these
- (vii) Identify the measurability of $\sup(f, g)$ when f and g are measurable.
- a) Measurable
b) Not measurable
c) Continuous
d) None of these
- (viii) Write the integral of a bounded measurable function over a finite measure set.
- a) Finite
b) Infinite
c) Undefined
d) None of these
- (ix) Write the integral of a measurable nonnegative function.
- a) Defined in $[0, \infty]$
b) Always finite
c) Always zero
d) None of these
- (x) Write the integral of a function over $\mathbb{R}$ of finite measure.
- a) Finite
b) Infinite
c) Undefined
d) Zero
- (xi) Select the correct relationship between convexity and absolute continuity.
- a) Convex functions are absolutely continuous on compact intervals
b) Convex functions are nowhere absolutely continuous

- c) Convexity implies monotonicity
(xii) Select the correct property of total variation.
a) Negative
c) Infinite for monotone functions
(xiii) Select the correct implication of absolute continuity.
a) Uniform continuity on compact intervals
c) Infinite variation
(xiv) Select the correct statement about $L^p(\mu)$ for $1 \leq p < \infty$.
a) It is not a normed space
b) It is a Banach space
c) It is incomplete
d) None of these
(xv) Select the correct expression for convergence in L^p .
a) $\|f_n - f\| \rightarrow 0$
b) $f_n(x) \rightarrow f(x)$
c) $\sup|f_n - f| \rightarrow 0$
d) None of these

Group-B

(Short Answer Type Questions)

3 x 5=15

2. Explain Lebesgue's Differentiation Theorem. (3)
3. Let f and g be non-negative measurable functions. Then examine $\int f dx + \int g dx = \int (f + g) dx$. (3)
4. Define Lebesgue measurable set and examine that if $m^*(E) = 0$, then E is measurable. (3)
5. Show that if f is measurable, then $\{x: f(x) = \alpha\}$ is measurable for each extended real number α . (3)
6. Justify that every convex function on an open interval is continuous. (3)

OR

Justify that if $\mu(X) < \infty$ and $0 < p < q \leq \infty$, then $L^q(\mu) \subseteq L^p(\mu)$. (3)

Group-C

(Long Answer Type Questions)

5 x 6=30

7. Examine that outer measure of an interval equals its length. (5)
8. Let f be a continuous function and g be a measurable function. Show that the composite function $f \circ g$ is measurable. (5)
9. Explain Lebesgue's dominated convergence theorem. (5)
10. If f is integrable on (a, b) and $\int_a^x f dx = 0$ for all $x \in (a, b)$, then deduce that $f = 0$ almost everywhere in (a, b) . (5)
11. Justify that $L^p(\mu)$ is complete for $1 \leq p \leq \infty$. (5)

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12. Write the definition of weak convergence in $L^p(\mu)$. (5)

OR

Write the definition of the L^p norm and justify that it satisfies the properties of a norm. (5)

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