



45610

**BRAINWARE UNIVERSITY****Term End Examination 2025-2026****Programme – M.Sc.(MATH)-2025****Course Name – Functional Analysis****Course Code - MSM20502****(Semester II)**

Library
Brainware University
 398, Ramkrishnapur Road, Barasat
 Kolkata, West Bengal-700125

Full Marks : 60**Time : 2:30 Hours**

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group-A

(Multiple Choice Type Question)

1 x 15=15

1. Choose the correct alternative from the following :

- (i) Select the correct statement.
- a) Every metric space is a normed space b) Every normed space is a metric space
 c) Every vector space is metric space d) Every metric space is a vector space
- (ii) Select the correct answer. A normed space is a Banach Space if and only if
- a) Every sequence is convergent b) Every Cauchy sequence is convergent
 c) Every convergent sequence is Cauchy d) None of these
- (iii) Select the correct answer. Let X be a finite dimensional normed space. Then two normed on X are
- a) equivalent b) incomparable
 c) equivalent if two normed are complete d) None of these
- (iv) Select the correct answer. Continuous image of a compact set under a mapping between two metric spaces is:
- a) always compact b) sometimes compact
 c) never compact d) always closed but not bounded
- (v) Select the correct answer. A mapping between two metric spaces from X to Y is called a closed map if:
- a) Image of every closed set of X is closed in Y b) Image of every subsets of X is closed in Y
 c) Pre-image of every closed set of Y is open in X d) Pre-image of every subsets of Y is closed in X
- (vi) Write the correct answer. The inverse of a closed linear operator is always
- a) closed b) closed if the operator is bounded
 c) closed if domain is closed d) closed if domain is complete
- (vii) Write the correct answer. The set of all real number R is
- a) first category b) second category
 c) countable d) other
- (viii) Write the correct answer. In an NLS every bounded linear functional is always
- a) continuous b) discontinuous
 c) continuous on the bounded subsets d) other

Justify that the space $l^p, p \neq 2$ is not a Hilbert space.

(3)

Group-C
(Long Answer Type Questions)

5 x 6=30

7. Analyze that the inner product of an inner product space X is continuous on $X \times X$. (5)
8. Let X be a normed linear space over a field of scalars F . Then examine that every Cauchy sequence in X is bounded (5)
9. Explain the Hahn-Banach theorem for complex normed vector spaces. (5)
10. Write and prove the Banach-Steinhaus theorem (Uniform Boundedness Principle). (5)
11. Define a metric space. Give three examples of metric spaces, including at least one from a function space. (5)
12. Justify if the set of vectors $\{(2,1,1), (1,2,2), (1,1,1)\}$ is linearly dependent in $\mathbb{R}^3$. (5)

OR

Let $T: X \rightarrow Y$ be a linear operator on X . Justify that if T is a bounded linear operator, then the image $T(S)$ of the unit sphere $S = \{x \in X: \|x\| = 1\}$ is bounded.

(5)

Library
Brainware University
398, Ramkrishnapur Road, Barasat
Kolkata, West Bengal-700125