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BRAINWARE UNIVERSITY

Term End Examination 2025-2026
Programme – M.Sc.(MATH)-2025
Course Name – Integral Equations & Calculus of Variations
Course Code - MSM20503
(Semester II)

Full Marks : 60 **Time : 2:30 Hours**

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group-A
(Multiple Choice Type Question) 1 x 15=15

1. *Choose the correct alternative from the following :*

(i) Identify the option that is an example of symmetric kernel

a) $(x-t)$
b) $(x+t)$
c) Both (a) and (b)
d) None of these.

(ii) For the kernel $K(x,t)$, if $K(x,t)=K(x-t)$, then $K(x,t)$ can be identified as

a) Separable Kernel
b) Degenerate Kernel
c) Difference Kernel
d) None of these.

(iii) Choose the correct option: Initial value problem is always converted into

a) Fredholm integral equation
b) Volterra integral equation
c) Both (a) and (b)
d) None of these.

(iv) Identify the definition of Resolvent Kernel?

a) $y(x)=f(x)+\lambda\int_a^b R\,(x,t;\lambda)f(t)dt$
b) $y(x)=f(x)+\lambda\int_a^x \Gamma\,(x,t;\lambda)\,f(t)dt$
c) Both (a) & (b)
d) Neither (a) nor (b)

(v) Choose the correct option. One of the eigen functions corresponding to the eigen value $\lambda=\frac{4}{\pi}$ of the integral equation $y(x)=\lambda\int_0^{\pi}(\cos^2x.\cos2t+\cos3x.\cos^3t)\,y(t)dt$ is

a) $\cos^2x$
b) $\cos3x$
c) $\cos2t$
d) $\cos^3t$

(vi) Identify the kernel for the integral solution $\phi(x) = x^2 + \int_0^x e^{t-x}\, \phi(t)\, dt$

a) e^{t+x}

b) e^{x-t}

c) e^{t-x}

d) None of these

(vii) Select the correct option. The function $K(x,y)$ is said to be symmetric if

a) $K(x,y) = K(y,x)$

b) $K(x,y) = -K(y,x)$

c) $K(x,y) = 0$

d) None of these

(viii) The equation $\int_0^x \frac{\phi(y)}{(x-y)^a} dy = f(x), (0 < \alpha < 1)$ can be justified as

a) Fredholm equation

b) Abel equation

c) Maxwell equation

d) Picard's equation

(ix) Let $f(x)$ be a periodic function with period L then for all x, then conclude the correct option.

a) $f(x+L) = f(x)$

b) $f(x+2L) = 2f(x)$

c) $f(Lx) = Lf(x)$

d) None of these

(x) Determine the value of α for which the integral equation $u(x) = \alpha \int_0^1 e^{x-t} u(t) dx$, has a non–trivial solution is

a) -2

b) -1

c) 1

d) 2

(xi) Identify the kernel of the integral equation $u(x) = x + \int_0^{\frac{1}{2}} u(t) dt$ (Fredholm Integral equation of 2nd kind)

a) 0

b) 3

c) 2

d) 1

(xii) Identify the even function of t from the following options

a) t^2

b) $t^2 - 4t$

c) $t^3 + 6$

d) None of these

(xiii) Choose the correct option. Geodesics on a surface is

a) curve along which distance between two points on the surface is minimum.

b) curve along which distance between two points on the surface is maximum

c) both (a) & (b)

d) None of these.

(xiv) Isoperimetric problem can be classified as

a) finding a closed curve of given length which encloses minimum area.
b) finding a closed curve of any length which encloses maximum area.
c) finding a closed curve of given length which encloses maximum area.
d) None of these.

(xv) Evaluate the shortest distance between the lines (x–3)/3 = (y –8)/(–1) = (z–3)/1,

(x + 3)/(–3) = (y+7)/2 = (z–6)/4 is

a) 1
b) 3
c) 3√30
d) Other

Group-B

(Short Answer Type Questions) 3 x 5=15

2. Illustrate different types Fredholm Integral Equation. (3)

3. State and prove the linearity property of the Fourier transformation. (3)

4. Solve $f'(x) = x + \int_0^x f(x-t)\cos t\,dt,\ f(0) = 4$ (3)

5. Justify that the functional $\int_0^1 (xy + y^2 - 2y^2y')dx,\ \ y(0) = 1,\ \ y(1) = 2$ cannot have any stationary function. (3)

6. Support the use of isoperimetric constraints in variational problems with an example. (3)

OR

Validate the variational nature of the geodesic problem. (3)

Group-C

(Long Answer Type Questions) 5 x 6=30

7. Express the differential equation with the given initial conditions: (5)
$y'' + (x^2 - x)y = 1,$
$y(0) = 1 = y'(0)$ into an integral equation.

8. Calculate the eigen values and eigen functions of the homogeneous integral equation (5)

$$y(x) = \lambda \int_0^1 e^x e^t y(t)dt$$

9. Illustrate that the Laplace transformation satisfies the shifting formula. (5)

10. Deduce the value of $u(x)$ from the integral equation: $\int_a^x \frac{u(t)dt}{\sqrt{x-t}} = f(x)$ (5)

11. Apply isoperimetric method to maximize area enclosed by a curve of fixed length. (5)

12. Evaluate the extremal of the functional $J\,[y] = \int_0^2 \frac{\sqrt{(1+y'^2)}}{2x}$ (5)

OR

Evaluate the Laplace transformation of $F(t) = \sin at$ by definition. (5)

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