BRAINWARE UNIVERSITY

Term End Examination 2023-2024
Programme – M.Sc.(MATH)-2022
Course Name – Applied Numerical Analysis
Course Code - MSCMC402
(Semester IV)

Full Marks : 60 **Time : 2:30 Hours**

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group-A
(Multiple Choice Type Question) 1 x 15=15

1. *Choose the correct alternative from the following :*

(i) Identify the correct option. The Jacobi's method is a method of solving a matrix equation on a matrix that has no zeroes along ______

a) Leading diagonal
b) Last column
c) Last row
d) Non-leading diagonal

(ii) Identify the correct option. In Jacobi's method the elements of the orthogonal matrix S_1 are defined as

a) $s_{ij} = -\sin\theta, s_{ji} = \sin\theta, s_{ii} = \sin\theta, s_{jj} = \cos\theta$
b) $s_{ij} = -\sin\theta, s_{ji} = \sin\theta, s_{ii} = \cos\theta, s_{jj} = \cos\theta$
c) $s_{ij} = -\sin\theta, s_{ji} = \sin\theta, s_{ii} = \cos\theta, s_{jj} = -\cos\theta$
d) None of these

(iii) Select the correct option. A matrix A can be factorized as a product of a lower triangular matrix and an upper triangular matrix, if

a) A is non-singular
b) all principle minors are non-zero
c) A is only symmetric
d) none of these

(iv) Select the correct option. Let us consider a square matrix A of order n with Eigen values of a, b, c then the Eigen values of the matrix A^T could be

a) a, b, c
b) -a, -b, -c
c) a-b, b-a, c-a
d) a^{-1}, b^{-1}, c^{-1}

(v) The Eigen values of a 3×3 matrix is $\lambda_1, \lambda_2, \lambda_3$ then the Eigen values of a matrix A^3 are __________. Predict the correct option

a) $\lambda_1, \lambda_2, \lambda_3$

b) $\frac{1}{\lambda_1}, \frac{1}{\lambda_2}, \frac{1}{\lambda_3}$

c) $\lambda_1^3, \lambda_2^3, \lambda_3^3$

d) $\frac{1}{\lambda_1^3}, \frac{1}{\lambda_2^3}, \frac{1}{\lambda_3^3}$

(vi) Identify the Eigenvalues of the matrix $\begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$.

a) $2+\sqrt{2}, 2-\sqrt{2}, 2$

b) $2+\sqrt{3}, 2-\sqrt{3}, 2$

c) 2, 2, 2

d) 3, 2, 1

(vii) Choose the correct option. The predictor-corrector method is a combination of __________

a) midpoint and trapezoidal rules

b) backward Euler method and Trapezoidal rule

c) implicit and explicit methods

d) forward and backward Euler methods

(viii) Relate the correct answer. The second-order Runge-Kutta method uses __________ as a predictor.

a) backward order method

b) forward Euler method

c) midpoint rule

d) multipoint method

(ix) Select the correct option. The first two steps of the fourth-order Runge-Kutta method use __________

a) Euler methods

b) Forward Euler method

c) Backward Euler method

d) Explicit Euler method

(x) Select the correct option. The characteristic curves for an elliptic system are __________

a) real and imaginary

b) both real

c) both imaginary

d) both zeros

(xi) Select the correct option .The Crank-Nicolson scheme is __________

a) fourth-order accurate

b) third-order accurate

c) second-order accurate

d) first-order accurate

(xii) Errors may occur in performing numerical computation on the computer due to some specific reasons. Select the correct reason from the followings.

a) Rounding errors
b) Power fluctuation
c) Operator fatigue
d) Back substitution

(xiii) Choose the correct option. The ordinary differential equations are solved numerically by__.

a) Euler method
b) Taylor method
c) Runge-Kutta method
d) All of these

(xiv) Select the correct statement that is true for elliptic equations.

a) The solution at all points must be carried out simultaneously
b) The solution can be marched from some initial conditions
c) The solution can be approximated in some of the points
d) The solution process should be carried out simultaneously for some region and then marching can be done

(xv) Choose the correct method, ______ is defined as

$$\frac{dy}{dx} = f(x,y), y(x_0) = y_0, y^{n+1}(x) = y_0 + \int_{x_0}^{x} f(x, y^n)\,dx$$

a) Taylor's series method
b) Picard's method
c) Euler's method
d) modified Euler's method

Group-B

(Short Answer Type Questions) 3 x 5=15

2. The product of two Eigenvalues of the matrix $A = \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$ is 16. Identify the third eigenvalue of A. (3)

3. State Cayley-Hamilton theorem and identify the characteristic equation of the matrix $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$. (3)

4. Apply Runge-Kutta fourth order method to compute an approximate value of y when $x = 0.2$ given that $\frac{dy}{dx} = x + y$ and $y = 1$ when $x = 0$ by taking the spacing length $h = 0.2$. (3)

5. Conclude that the region such that the following equation $x^3 \frac{\partial^2 u}{\partial x^2} + 27 \frac{\partial^2 u}{\partial y^2} + 3 \frac{\partial^2 u}{\partial x \partial y} + 5u = 0$ acts as an elliptic equation is $x > \left(\frac{1}{12}\right)^{\frac{1}{3}}$. (3)

6. Deduce an approximate series solution of the simultaneous equations (3)

$\frac{dx}{dt} = xy + 2t, \ \frac{dy}{dt} = 2ty + x$ subject to the initial condition

$x = 1, y = -1, t = 0.$

OR

State and explain the necessary and sufficient condition for the stability of a finite difference method to solve a PDE. (3)

Group-C

(Long Answer Type Questions) 5 x 6=30

7. Calculate the value of $y(0.20)$ for the initial value problem $\frac{dy}{dx} = y^2 \sin x$ with $y(0) = 1$ using Milne's predictor-corrector method, taking $h = 0.05, y_1 = 1.001251, y_2 = 1.005021,$ $y_3 = 1.011356.$ (5)

8. Given $A = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$. Estimate the eigenvalues and corresponding eigenvectors of the given matrix using Power method. (5)

9. Evaluate the value of $y(0.2)$ by solving the ODE $y' = y + e^x, y(0) = 0$ using modified Euler's method. (5)

10. Explain the Strum sequence for finding eigenvalues of a tri-diagonal matrix. (5)

11. Apply RK4 method and evaluate $y(0.2)$ by solving the differential equation $\frac{dy}{dx} = x + y, y(0) = 1.$ (5)

12. Evaluate the solution of the equation $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$ with the boundary conditions $u = 0$ at $x = 0$ and $1, t > 0$ and the initial conditions $u = \frac{1}{2} \sin \pi x, \frac{\partial u}{\partial t} = 0,$ when $t = 0, 0 \leq x \leq 1$, for x=0.2 and $t = 0.1$. (5)

OR

Use the Crank-Nicolson method to calculate a numerical solution of the problem $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \ 0 < x < 1, t > 0$ where $u(0,t) = u(1,t) = 0, t > 0, u(x,0) = 2x,$ $t = 0$. Evaluate that the value of $u\left(\frac{1}{2}, \frac{1}{8}\right) = \frac{2}{3}$ by taking $h = \frac{1}{2}$ and $k = \frac{1}{8}$. (5)

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