BRAINWARE UNIVERSITY

Term End Examination 2023-2024
Programme – M.Sc.(MATH)-2022
Course Name – Mathematical Ecology
Course Code - MSCME402B
(Semester IV)

Full Marks : 60 **Time : 2:30 Hours**

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group-A

(Multiple Choice Type Question) 1 x 15=15

1. *Choose the correct alternative from the following :*

(i) Identify the correct option, A Malthusian growth model sometimes called it in the name of.....

a) Logistic growth model. b) SIR model
c) SI model d) Simple exponential growth model

(ii) Identify the general solution of $x(t)$ of the system $x' = -x + 2y, y' = 4x + y$ is given by, here c_1 *and* c_2 are arbitrary constant.

a) $c_1e^{3t} + c_2e^{-3t}$ b) $c_1e^{2t} + c_2e^{-3t}$

c) $c_1e^{2t} + c_2e^{4t}$ d) $c_1e^{3t} + c_2e^{-4t}$

(iii) Select the correct statement, In the Harvest model with optimal control theory, the term "yield" refer to.....

a) The total biomass of the harvested species. b) The monetary value of the harvested resource.
c) The rate of reproduction of the harvested species. d) The maximum sustainable harvest rate.

(iv) Choose the correct option, _____terms is often used interchangeably with "activator" in bio-mathematics.

a) Inhibitor b) Enhancer
c) Repressor d) Substrate

(v) Identify the jacobian matrix of the given system is $\frac{dx}{dt} = -x - y$

$\frac{dy}{dt} = x - y$

a) $J = \begin{bmatrix} -1 & -1 \\ 1 & -1 \end{bmatrix}$ b) $J = \begin{bmatrix} -2 & -1 \\ 1 & -1 \end{bmatrix}$

c) $J = \begin{bmatrix} -1 & -2 \\ 1 & -1 \end{bmatrix}$

d) $J = \begin{bmatrix} -1 & -1 \\ 5 & -1 \end{bmatrix}$

(vi) The Density dependence growth model may be express in the form of....., N(t) consider be the density of the species at time t

a) $\frac{dN}{dt} = rN - aN^2$

b) $\frac{dN}{dt} = r$

c) $\frac{dN}{dt} = -a$

d) none of these.

(vii) $\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right)$, where $N = Population\ density, r = intrinsic\ growth\ rate, K = carrying\ capacity$. The given model is classified as

a) Logistic growth model.

b) Malthus growth model.

c) Two species model.

d) none of these.

(viii) Identify the correct answer, the population doubling time for the exponentially growing species is

a) $\ln 2$

b) $\frac{\ln 2}{r}$

c) r

d) None of these

(ix) The Gompertz growth model express as

a) $\frac{dN}{dt} = r_0 e^{-\theta t} N$

b) $\frac{dN}{dt} = r_0 e^{\theta t} N$

c) $\frac{dN}{dt} = e^{-\theta t} N$

d) none of these.

(x) Choose the correct option,

$$\frac{dx}{dt} = x(\alpha - \beta y)$$
$$\frac{dy}{dt} = y(-\gamma + \delta x)$$

The number of equilibrium point of the above model.

a) 1

b) 2

c) 3

d) 4

(xi) The relation between Latent Period and incubation Period express as, write the correct option.

a) Latent Period < Incubation Period

b) Latent Period>Incubation Period

c) Latent Period≤Incubation Period

d) None of these

(xii) The system represented as $\frac{dB}{dT} = rB\left(1 - \frac{B}{K}\right) - \frac{mB^2}{a^2+B^2}$ for Nondimensionalization we apply $B = xa$ and $T = \frac{a}{m}t$ then the system becomes $\frac{dx}{dt} = px\left(1 - \frac{x}{q}\right) - \frac{x^2}{1+x^2}$ then evaluate the value of p is.

a) $\frac{ar}{m}$

b) $\frac{a}{m}$

c) $\frac{r}{m}$

d) None of these

(xiii) Select the correct option, $x_{t+1} = f(x_t)$ be the discrete model, f' is continuous and $\bar{x}$ be the equilibrium point of then system is unstable if

a) $|f'(\bar{x})| > 1$ b) $|f'(\bar{x})| < 1$

c) $|f'(\bar{x})| = 1$ d) None of these

(xiv) The model is classify as

$$\frac{dN_1}{dt} = r_1 N_1 \left(1 - \frac{N_1}{K_1} + \frac{a_{12}}{K_1} N_2\right)$$

$$\frac{dN_2}{dt} = r_2 N_2 \left(1 - \frac{N_2}{K_2} + \frac{a_{21}}{K_2} N_1\right)$$

a) Single species competition model b) Two species competition model

c) Three species competition model d) None of these

(xv) Identify the critical point of the model $\frac{dx}{dt} = x - ax^2$ from the given option.

a) 1 b) $\frac{1}{a}$

c) 2 d) None of these

Group-B

(Short Answer Type Questions) 3 x 5=15

2. Examine the general solution of the linear autonomous system (3)

$$\frac{dx}{dt} = y$$

$$\frac{dy}{dt} = -x$$

3. Describe Stability of single species model. (3)

4. Express a model of three species with holling type-II functional response. (3)

5. Summarize harvesting in fishing cultivation. (3)

6. Estimate Quasi asymptotic Stability. (3)

OR

Estimate Asymptotic stability. (3)

Group-C

(Long Answer Type Questions) 5 x 6=30

7. Show that the critical point (0,0) of the given system is asymptotically stable, (5)

$$\frac{dx}{dt} = -2x$$

$$\frac{dy}{dt} = -4y$$

8. Explain the stability of the single species logistic model. (5)

9. Calculate the equilibrium point and discuss the stability around K_0. (5)

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right)\left(\frac{N}{K_0} - 1\right)$$

10. Explain the stability of the equilibrium point (1,0) and draw the phase plane: (5)

$$\frac{dx}{dt} = x(1-x) - xy$$

$$\frac{dy}{dt} = \beta y(x - \alpha)$$

Where α *and* β *are positive constant.*

11. Construct the Jacobian matrix of the given model: (5)

$$\frac{dN}{dt} = r_1 N\left(1 - \frac{N}{K}\right) - \frac{a_1 N P_1}{b_1 + N} - \frac{a_2 N P_2}{b_2 + N}$$

$$\frac{dP_1}{dt} = \frac{e_1 a_1 N P_1}{b_1 + N} - d_1 P_1$$

$$\frac{dP_1}{dt} = \frac{e_2 a_2 N P_2}{b_2 + N} - d_2 P_2$$

12. Construct the mutualistic model two competitive species. (5)

OR

Explain the two prey one predator model of holling type-II model. (5)

****