BRAINWARE UNIVERSITY

Coursework Examination 2018 – 19 (June 2019)

Programme – Doctorate of Philosophy in Mathematics

Course Name – Fuzzy Sets and Their Applications

Course Code – PHD-MAT-02

Time allotted: 4 Hours   **Full Marks : 100**

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group –A

(Objective Type Questions)   **10 x 1 = 10**

1. *Answer the following***:**

(i) *Choose the correct option:*
    The truth values of traditional set theory is _____ and that of fuzzy set is _____
    (a)   Either 0 or 1, between 0 & 1   (b)   Between 0 & 1, either 0 or 1
    (c)   Between 0 & 1, between 0 & 1   (d)   Either 0 or 1, either 0 or 1

(ii) Express the statement "integers close to 10" as a fuzzy set.

(iii) *Choose the correct option:*
    A continuous super idempotent t-conorm is called ------------
    (a) Strictly Archimedean   (b) Archimedean   (c) t-norm   (d) None of these

(iv) *Choose the correct option:*
    The boundary conditions satisfied by the standard fuzzy complement are
    (a) $c(0) = 1$ and $c(1) = 1$ (b) $c(0) = 0$ and $c(1) = 1$
    (c) $c(0) = 0$ and $c(1) = 0$ (d) $c(0) = 1$ and $c(1) = 0$

(v) State Zadeh's extension principle.

(vi) State the compositional rule of inference.

(vii) Define Cauchy sequence in a fuzzy metric space.

(viii) *Choose the correct option:*
    For standard fuzzy intersection, which of the following hold?
    (a) $i(a,b) = min(a,b)$ (b) $i(a,b) = ab$ (c) $i(a,b) = a-b$ (d) None

(ix) Give an example of a quantified fuzzy proposition.

(x) *Choose the correct option:*
How is fuzzy logic different from conventional control methods?
(a) IF and THEN approach (b) FOR approach
(c) WHILE approach (d) DO approach

Group – B

(Short Answer Type Questions) 6 x 5 = 30

Answer any *six* **from the following:**

2. State and prove the First Decomposition Theorem. 5

3. What are the roles of $\propto$-cuts and strong $\propto$-cuts in fuzzy set theory? Describe these concepts with suitable example. 5

4. Define t-norm. Show that the standard fuzzy intersection is the only idempotent t-norm. 5

5. Give an example for a reflexive transitive relation and verify. 5

6. What is fuzzy equivalence relation? Discuss with example. 5

7. Let $(X, M, *)$ be a compact fuzzy metric space and T a continuous self-map of X for all $x, y \in X$ satisfying
$$F(M(x, y, t), M(Tx, x, t), M(Tx, y, t), M(Tx, Ty, t)) < 0 \text{ where } F \in \mathfrak{F}$$
Then, T has a unique fixed point in X. 5

8. What is R-implication? Discuss with four well-known examples. 5

9. Discuss a predominant method of defuzzification as used in fuzzy control. 5

Group – C

(Long Answer Type Questions) 6 x 10 = 60

Answer any *six* **from the following:**

10. State the extension principle by which any crisp function can be fuzzified. Discuss any three properties that hold for functions fuzzified by the extension principle with proof.

10

11. Consider the fuzzy sets A, B and C defined on the interval $X = [0,10]$ of real numbers by the membership grade functions
$$A(x) = \frac{x}{x + 2}, \quad B(x) = 2^{-x}, \quad C(x) = \frac{1}{1 + 10(x - 2)^2} .$$
Determine mathematical formulae and graphs of the membership grade functions of each of the following sets:
(i) $\bar{A}, \bar{B}, \bar{C}$ (ii) $A \cup B, A \cup C, B \cup C$ (iii) $A \cap B, A \cap C, B \cap C$

10

12. State the First and Second Characterization Theorems of Fuzzy Complements. Provide a proof of the Second Characterization Theorem.

10

13. Write short notes on the following:
    (a) Fuzzy partial ordering
    (b) Fuzzy morphism

10

14. (a) What is fuzzy metric space?

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(b) Let (X, d) be a complete metric space on X. Suppose the mapping $T : X \rightarrow X$ satisfy the contractive condition, $(Tx,) < k\ (x,)$ for all $x, \in X$, $k \in [0,1)$. If T has a unique fixed point in X with respect to the metric (X, d), then T has a unique fixed point with respect to the induced fuzzy metric.

8

15. Solve the following fuzzy relation equation for the max-min composition:
$$Po \begin{bmatrix} .9 & .7 & .2 \\ .8 & 1 & .5 \\ .1 & .3 & .6 \end{bmatrix} = [.8\ .7\ .5]$$

10

16. What is fuzzy implication? Give an overwiew of the specific operators of fuzzy implications that are commonly used in literature.

10

17. What is fuzzy controller? Draw a schematic diagram of a fuzzy controller. Discuss the step of fuzzification.

10