BRAINWARE UNIVERSITY

Coursework Examination 2018 – 19 (June 2019)

Programme – Doctorate of Philosophy in Mathematics

Course Name – Fixed Point Theory and Applications

Course Code – PHD-MAT-03

Time allotted: 4 Hours   **Full Marks : 100**

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group –A

(Objective Type Questions)   10 x 1 = 10

1.   *Answer the following :*

(i)   Lipschitz constants are always greater than one. (*True/ False*)

(ii)   Compatible mappings are always commuting (*True/ False*)

(iii) Kannan type mappings may not be always continuous. (*True/ False*)

(iv)   If the diameter of a set is zero, then the set is a null set. (*True/ False*)

(v)   *Choose the correct option:*

Which of the following inequality is used to prove Banach Fixed Point Theorem?

(a) $d(f_x, f_y) \leq kd(x,y), k = 0,1$ (b) $d(f_x, f_y) \geq d(x,y)$

(c) $d(f_x, f_y) \geq kd(x,y), k \in (0,1)$ (d) $d(f_x, f_y) \leq kd(x,y), k \in (0,1)$

(vi) *Choose the correct option:*

Contraction mapping introduced by Sehgal and Bharucha-Reid is known as

(a) q-contraction   (b) C-contraction   (c) φ-contraction   (d) g-contraction

(vii) *Choose the correct option:*

The famous Banach Fixed Point Theorem came into literature in the year

(a)   1920   (b) 1921   (c) 1922   (d) 1923

(viii) *Choose the correct option:*

Altering distance function in metric space was first used by

(a) Goebel and Kirk   (b) Khan, Swaleh and Sessa   (c) Chowdhury and Das

(d) Meir and Keeler

(ix) Coupled Fixed Point Theorem was introduced by ……………………..

(x) *Choose the correct option:*

Let (X, d) be a metric space and $f, g : X \to X$ be two self-mappings. The mappings f and g are said to be commuting if

(a) $f^2 = g^2$ (b) $f^2 = g$ (c) $f = g^2$ (d) $fg = gf$

Group – B

(Short Answer Type Questions)   6 x 5 = 30

Answer any *six* from the following:

2. What do you mean by mixed monotone property in ordered metric space?   5

3. Define Cauchy sequence in fuzzy metric spaces. Distinguish between G-Cauchy sequence and M-Cauchy sequence.   5

4. Define commuting mappings and compatible mappings.   5

5. What do you mean by α-admissible and dominating mappings?   5

6. Discuss the significance of altering distance function in fixed point theory.   5

7 What do you mean by Hausdorff metric? Is it a metric?   5

8. Give an example of 2-metric space. Justify the example with the definition.   5

9. What is the difference between metric space and 2-metric space?   5

Group – C

(Long Answer Type Questions)   6 x 10 = 60

Answer any *six* from the following:

10. Elaborate the application of fixed point theory for solving a differential equation.   10

11. State and prove the Fixed Point Theorem due to Kannan.   10

12. Give a brief historical description of Altering distance function.   10

13. Discuss about the stability of the iteration process due to Ishikawa.   10

14.   Write short notes on the following :   10
         (a) Menger Probabilistic metric space
         (b) KM-Fuzzy Metric Space
         (c) GV-Fuzzy Metric Space

15.   Define D-metric space. What is the drawback of this kind of spaces? Discuss in   10
       details with remedial measures.

16.   What do you mean by coupled coincidence point? "Coincidence point is always a   10
       fixed point". Justify the statement.

17.   Discuss the development of compatible mapping from commuting mapping.   10