BRAINWARE UNIVERSITY

Term End Examination 2018 - 19

Programme– B.Tech.(CSE) / B.Tech.(ECE)

Course Name – Linear Algebra & Differential Equations

Course Code –BSC(CSE)201 / BSC(ECE)201

(Semester – 2)

Time allotted: 3 Hours **Full Marks: 70**

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group –A

(Multiple Choice Type Questions) 10 x 1 = 10

1. ***Choose the correct alternative from the following :***

(i) Gauss Elimination method reduces the coefficient matrix into a/an ____________ matrix.

a. diagonal
b. upper triangular
c. lower trianglar
d. symmetric

(ii) The matrix $\begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 1 \\ 4 & 5 & -3 \end{bmatrix}$ is

a. singular
b. non-singular
c. invertible
d. both b. and c.

(iii) For the linear transformation $T: V \to W, Rank(T) =$

a. $\dim(V)$
b. $\dim(W)$
c. $\dim(Ker(T))$
d. $\dim(Im(T))$

(iv) The transformation $T: R \to R$ defined by $T(x) = sinx$ is

a. linear
b. non-linear
c. neither a. nor b.
d. both a. and b.

(v) The eigenvalues of the matrix $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ are

a. 0, 2
b. 1, 1
c. $-1, -1$
d. none of these

(vi) For any two vectors $\bar{u}, \bar{v}$ in a Euclidean space,

a. $|\langle \bar{u}, \bar{v} \rangle| = \|\bar{u}\|\|\bar{v}\|$
b. $|\langle \bar{u}, \bar{v} \rangle| \geq \|\bar{u}\|\|\bar{v}\|$
c. $|\langle \bar{u}, \bar{v} \rangle| \leq \|\bar{u}\|\|\bar{v}\|$
d. none of these

(vii) The orthogonal trajectory of the hyperbola $xy = c$ is

a. $x^2 - y^2 = C$
b. $x^2 + y^2 = C$
c. $x^2 = Cy^2$
d. none of these

(viii) The transformed equation of

$$x^2 \frac{d^2y}{dx^2} + 2x\frac{dy}{dx} + 2y = 10\left(x + \frac{1}{x}\right)$$

is

a. $\frac{d^2y}{dz^2} + 2\frac{dy}{dz} + 2y = 10(e^z + e^{-z})$
b. $\frac{d^2y}{dz^2} + \frac{dy}{dz} + 2y = 10(e^z + e^{-z})$
c. $\frac{d^2y}{dz^2} + \frac{dy}{dz} + y = 10(e^z + e^{-z})$
d. none of these

(ix) The partial differential equation $xu_x + yu_y = u^2$ is

a. linear
b. non-linear
c. quasi-linear
d. none of these

(x) The equation $u_{tt} + u_{xx} = 0$ is of ____________ type.

a. elliptic
b. parabolic
c. hyperbolic
d. none of these

Group – B

(Short Answer Type Questions) 3 x 5 = 15

Answer any *three* from the following :

2. Using the idea of the rank of a matrix, examine if the system of equations 5

$$3x + y - 5z + 1 = 0$$
$$x - 2y + z + 5 = 0$$
$$x + 5y - 7z = 2$$

is consistent or not.

3. Show that the set $B = \{(1,2,1), (0,1,0), (0,0,1)\}$ is a basis of R^3 . Hence express the vector $(1,2,3)$ as a linear combination of the basis vectors. 5

4. Show that the matrix $A = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$ is not diagonalizable. 5

5. Solve: $\left(y^2 e^{xy^2} + 4x^3\right)dx + \left(2xye^{xy^2} - 3y^2\right)dy = 0$. Hence find the particular solution if $y = 0$ at $x = 0$. 5

6. Using the method of separation of variables, solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$ where $u(x,0) = 6e^{-3x}$. 5

Group – C

(Long Answer Type Questions) 3x 15 = 45

Answer any *three* from the following :

7. (a) Solve the following system of equations by Cramer's Rule: 5

$$x - 3z = 1$$
$$2x - y - 4z = 2$$
$$y + z = 4$$

(b) Show that the system of equations 8

$$2x - 2y + z = \lambda x$$
$$2x - 3y + 2z = \lambda y$$
$$-x + 2y = \lambda z$$

can possess a non-trivial solution only if $\lambda = 1$ and $\lambda = -3$. Obtain the general solution for $\lambda = -3$.

(c) Show that the matrix AA^T is always symmetric. 2

8. (a) Let $R_{2\times 2}$ be the set of all 2×2 matrices with real entries. Show that $R_{2\times 2}$ forms a vector space over R. 7

(b) A linear transformation $T: R^3 \to R^3$ is defined by 5

$$T(x_1, x_2, x_3) = (x_1 - x_2, x_1 + 2x_2, x_2 + 3x_3).$$

Find T^{-1}.

(c) If $T: V \to W$ be a linear transformation, then show that $Ker(T)$ is a subspace of V. 3

9. (a) Find the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{bmatrix}$. 9

(b) Use Gram-Schmidt process to convert the basis $\{(1,2,-2), (2,0,1), (1,1,0)\}$ of R^3 into an orthogonal basis and then extend it to an orthonormal basis. 6

10. (a) Use the method of variation of parameters to solve the equation 8

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = e^x \log x$$

(b) Solve the following system of equations: 7

$$\frac{dy}{dx} + 2y - 3z = x$$
$$\frac{dz}{dx} + 2z - 3y = e^{2x}$$

11. An infinitely long plane uniform plate is bounded by two parallel edges and an end at right angles to them. The breadth is π; this end is maintained at a temperature u_0 at all points and other edges are at zero temperature. Determine the temperature at any point of the plate in the steady state. 15
