BRAINWARE UNIVERSITY

Term End Examination 2018 - 19

Programme –Bachelor of Science (Honours) in Computer Science

Course Name -Algebra and Multivariable Calculus

Course Code - BCS203C

(Semester – 2)

Time allotted: 3 Hours   **Full Marks : 70**

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group –A

(Multiple Choice Type Questions)   10 x 1 = 10

1. *Choose the correct alternative from the following:*

(i) The remainder when 3^{10} is divided by 7 is

    a. 0   b. 1

    c. 7   d. 4

(ii) For Z_8, $[2].[7] =$

    a. $[14]$ b. $[6]$

    c. $[9]$ d. $[1]$

(iii) Number of edges in a complete graph with n-vertices is:

    a. $^nC_1$ b. $^nC_2$

    c. $^nC_3$ d. ^nC_n

(iv) If G is a tree with n vertices, then the number of edges of G are

    a. n(n+1)   b. n-1

    c. n   d. n(n-1)

(v) If $f_x(a,b) = f_y(a,b) = 0$, then (a,b) is a

    a. saddle point   b. extreme point

    c. critical point   d. isolated point

                                      e.

(vi)   If $f(x, y) = x^2 y$, then $df =$

    a. $2x^2 dx + dy$ b. $x - 2dy$

    c. $x + dy\,1$ d. $2xy\,dx + x^2 dy$

(vii) $\int_{0}^{1} \int_{y}^{\sqrt{y}} dx\,dy =$

    a. $\dfrac{1}{2}$ b. $\dfrac{1}{6}$

    c. $\dfrac{2}{3}$ d. $\dfrac{4}{3}$

(viii)   The value of the triple integral $\int_{0}^{1}\int_{5}^{8}\int_{0}^{9} dx\,dy\,dz$ is

    a. 25   b. 27

    c. 1   d. 3

(ix)   The general solution of $\dfrac{d^2 y}{dx^2} + 3\dfrac{dy}{dx} + 2y = 0$ is

    a. $y = Ae^{-x} + Be^{-2x}$ b. $y = e^{-x} + e^{-2x}$

    c. $y = e^{-x}(A + Bx)$ d. none of these

(x)   Integrating factor of the differential equation $\cos x \dfrac{dy}{dx} + y\sin x = 1$ is

    a. $\tan x$ b. $\cos x$

    c. $\sec x$ d. $\sin x$

Group – B

(Short Answer Type Questions)   3 x 5 = 15

Answer any *three* from the following :

2. State and prove Fermat's theorem.   [5]

3. Prove that a graph G has a spanning tree if and only if G is connected.   [5]

4. If $u = \log_e (x^3 + y^3 + z^3 - 3xyz)$, prove that $\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y} + \dfrac{\partial u}{\partial z} = \dfrac{3}{x + y + z}$.   [5]

5. Evaluate $\int_{0}^{2}\int_{0}^{z}\int_{0}^{x\sqrt{3}} \dfrac{x}{x^2 + y^2} dy\,dx\,dz$.   [5]

6. Solve: $(\sin x \cos y + e^{2x})dx + (\cos x \sin y + \tan y)dy = 0$.   [5]

Group – C

(Long Answer Type Questions) $3 \times 15 = 45$

Answer any *three* from the following :

7. (a) Find the remainder when $1! + 2! + 3! + \cdots\cdots\cdots + 50!$ is divided by 5.   [5]

    (b) Evaluate $\int\limits_{0}^{a} \int\limits_{0}^{\sqrt{a^2 - y^2}} (x^2 + y^2) dy dx$ by changing to polar co-ordinates.   [6]

    (c) If $z = u^2 + v^3$ where $u = \sin xy$ and $v = y^2$, find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$.   [4]

8. (a) Find the maximum value of $x^3 y^2$ subject to the constraint $x + y = 1$, using the method of Lagrange's multiplier.   [7]

    (b) Apply Dijkstra's method to find the shortest path and distance between the two vertices **A** & **G** in the given graph.   [8]
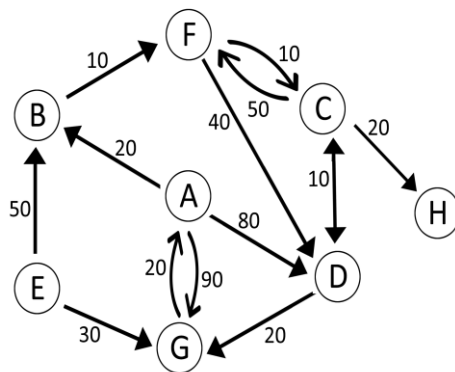
9. (a) Apply the method of variation of parameters to solve the equation: [6]

    $$\frac{d^2 y}{dx^2} + 9y = \sec 3x$$

    (b) Find two minimal spanning tree from the following graph using Prim's and Kruskal's algorithms.   [9]
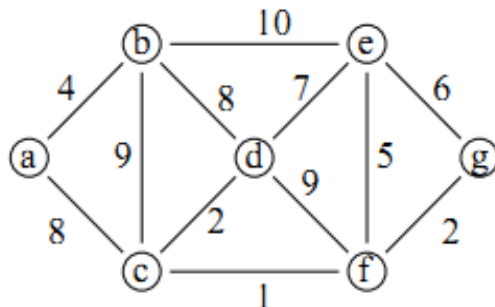
10. (a) Solve the Diophantine equations: 15x+7y=111 [5]

(b) Evaluate $\iiint_V (x^2 + y^2 + z^2) dxdydz$ where V is the volume of the cuboid [6]

bounded by the co-ordinate planes x=0, x=a, y=0, y=b, z=0, z=c

(c) Define Adjacency Matrix for a non-directed graph. Hence find the adjacency [4] matrix for the following graph.
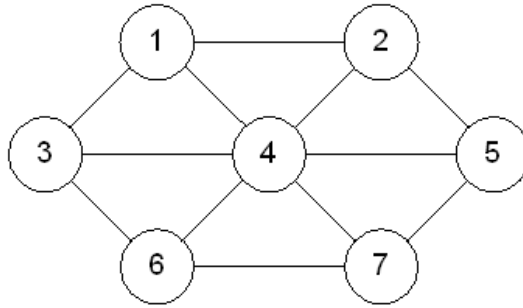
11. (a) If $\phi(v^2 - x^2, v^2 - y^2, v^2 - z^2) = 0$ where v is a function of x, y, z, show that [7]

$$\frac{1}{x} \cdot \frac{\partial v}{\partial x} + \frac{1}{y} \cdot \frac{\partial v}{\partial y} + \frac{1}{z} \cdot \frac{\partial v}{\partial z} = \frac{1}{v}.$$

(b) Evaluate $\iint dxdy$ over the domain bounded by $y = x^2$ and $y^2 = x$. [4]

(c) If gcd(a,b)=1, then prove that $\gcd(a+b, a^2 - ab + b^2) = 1$ or 3 [4]