BRAINWARE UNIVERSITY

Term End Examination 2023
Programme – M.Tech.(CSE)-AIML-2022
Course Name – Mathematics-II
Course Code - BSC-MMM201
(Semester II)

Full Marks : 60 **Time : 2:30 Hours**

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group-A
(Multiple Choice Type Question) 1 x 15=15

1. *Choose the correct alternative from the following :*

(i) Select which of the following is not linear transformation.

a) $T:R^2 \rightarrow R^2 : T(x,y) = (3x-y, 2x)$
b) $T:R^3 \rightarrow R^2 : T(x,y,z) = (3x+1, y-z)$
c) $T:R \rightarrow R^2 : T(x) = (5x, 2x)$
d) $T:R^3 \rightarrow R^2 : T(x,y,z) = (x,0,z)$

(ii) Select the correct option-A vector space V is finite dimensional if it has

a) <p style="text-align: left;"> finite basis
b) <p style="text-align: left;"> finite elements
c) <p style="text-align: left;"> no basis
d) <p style="text-align: left;"> none of these

(iii) Tell the integral $\int_{|z|=2} \frac{\cos z}{z^3} dz,$

a) πi
b) $-\pi i$
c) $2\pi i$
d) $-2\pi i$

(iv) Select the correct option-If f is a bounded entire function then f is

a) Non-constant
b) Not differentiable
c) Not analytic
d) Constant

(v) Euler lagrange equation formula is written as

a) $F_y - \frac{d}{dx}(F_y') = 0$
b) $F_y - \frac{d}{dx}(F_y'') = 0$
c) $F_x - \frac{d}{dx}(F_y') = 0$
d) None of these

(vi) Identify which of the following function f(z)satisfies Cauchy- Riemann equations?

a) $f(z) = \bar{z}$ at $z = 1 + i$
b) $f(z) = |z|^2$
c) $f(z) = \sqrt{|xy|}$
d) $f(z) = \frac{x^3(1+i)-y^3(1-i)}{x^2+y^2}, z \neq 0, f(0) = 0$

(vii) If f(x) =0 has a root between a & b, then examine f(a) & f(b) are of __________ signs.

a) Opposite
b) same
c) positive
d) negative

(viii) Compute the order of convergence of Newton Raphson Method is _________.

a) 1
b) 2
c) 3
d) 4

(ix) Let A and B are two $n \times n$ matrices. Select which of the following is equal to $trace(A^2B^2)$

a) $(trace(AB))^2$
b) $trace(AB^2A)$
c) $trace((AB)^2)$
d) $trace(BABA)$

(x) Select which of the following is not correct for analytic functions f(z) and g(z) in a region .

a) $f(z) = Re\, z$
b) $f(z) = Im\, z$
c) $f(z) = \cot z$
d) $f(z) = e^z$

(xi) Let V and W be vector spaces, and let $T : V \to W$ be linear. If $N(T) = \{0\}$, then select the correct option.

a) T is injective
b) T is surjective
c) T is bijective
d) Can not be decided

(xii)

$$A = \begin{pmatrix} 1 & 1 & -1 \\ -2 & 3 & 0 \\ -2 & 1 & 2 \end{pmatrix}$$

For the matrix, the express the eigen vector corresponding to the eigen value 3 is

a) (0,1,1)
b) (1,2,1)
c) (1,1,1)
d) None of these

(xiii) If $\lambda \neq 0$ is an Eigen value of a matrix A then evaluate $\det(A - \lambda I) =$

a) λ
b) $-\lambda$
c) 2λ
d) 0

(xiv) If $f(x, y) = x^2 + y^2$ then compute $f_{xy}(x, y) =$

a) 1
b) 0
c) 2
d) x+y

(xv) Solve $\lim_{(x,y)\to(0,0)} \frac{x^3 - y^3}{x - y} =$

a) 0
b) 1
c) 1/2
d) None of these

Group-B
(Short Answer Type Questions)

3 x 5=15

2. Tell the value of $\frac{1}{2\pi i} \oint \frac{\cos \pi z}{z^2 - 2} dz$, around a rectangle whose vertices are $2 \pm i, -2 \pm i$. (3)

3. Obtain the Euler's equation for the extremals of the functional (3)

$$\int_{x_1}^{x_2} \{y'^2\} dx$$

4. Evaluate the approximate error, relative error, percentage error in approximating 1/3 to 0.3333. (3)

5. Identify a basis and the dimension of the subspace W of R^3, where $W = \{(x, y, z) \in R^3 \mid x + y + z = 0\}$. (3)

6. Justify that the set $\left\{ \frac{1}{5}(3,0,4), \frac{1}{5}(-4,0,3), (0,1,0) \right\}$ is orthonormal. (3)

OR

Conclude the characteristic equation and Eigen values of the matrix $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ (3)

Group-C
(Long Answer Type Questions)

5 x 6=30

7. Describe Morera's theoem. (5)

8. Compute the extremal of the functional $\int_0^\pi (y'^2 - y^2)\, dx$ under the conditions $y(0) = 0$, $y(\pi) = 1$ and subject to the constraint $\int_0^\pi y\ dx = 1$ (5)

9. Evaluate the eigen values and the corresponding eigen vectors of the matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$. (5)

10. Evaluate f(3) using Newton's divided difference formula given that (5)

x	4	5	7	10	11	13
f(x)	48	100	294	900	1210	2028

11. Evaluate solutions of the system of equations by Matrix inversion method. (5)

x+3y+2z=17

x+2y+3z=16

2x-y+4z=13

12. Justify that any intersection of subspaces of a vector space V is a subspaces of V. (5)

OR

Let V be a vector space over a field of characteristic not equal to zero, let U and V be distinct vectors in V. Justify that $\{u, v\}$ is linearly independent iff $\{u + v, u - v\}$ is linearly independent. (5)

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