BRAINWARE UNIVERSITY

Term End Examination 2023
Programme – B.Tech.(CSE)-2018/B.Tech.(CSE)-2019/B.Tech.(CSE)-2020
Course Name – Linear Algebra and Differential Equations
Course Code - BSC(CSE)201
(Semester II)

Full Marks : 60 **Time : 2:30 Hours**

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group-A
(Multiple Choice Type Question) 1 x 15=15

1. *Choose the correct alternative from the following :*

(i) The general form of a first order linear equation in x is $\frac{dy}{dx}+Px=Q$, write the condition for P and Q

a) P and Q are both functions of x
b) P and Q are both functions of y
c) P and Q are the functions of x and y, respectively
d) P and Q are the functions of y and x, respectively

(ii) If $x^m y^n$ be the IF of the equation $(2ydx+3xdy)+2xy(3ydx+4xdy)=0$ then choose the value of m and n respectively

a) 1, 3
b) 2, 1
c) 2, 2
d) 1, 2

(iii) If the differential equation $\left(y+\frac{1}{x}+\frac{1}{x^2y}\right)dx+\left(x-\frac{1}{y}+\frac{A}{xy^2}\right)dy=0$ is exact, then choose the value of A

a) 2
b) 1
c) 0
d) -1

(iv) If $\lambda \neq 0$ is an Eigen value of a matrix A then identify the value of $\det(A-\lambda I)=$

a) λ
b) $-\lambda$
c) 2λ
d) 0

(v) dentify the expression of the vector (7,11) as a linear combination of the vectors (2,3) and (3,5)

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a) 1(2,3)+2(3,5)
b) 2(2,3)+1(3,5)
c) cannot be expressed
d) None of these

(vi) Select the correct option: If A is a non-null square matrix, then $A+A^T$ is a

a) symmetric matrix
b) skew-symmetric matrix
c) null matrix
d) none of these.

(vii) If $\lambda \neq 0$ is an Eigen value of a matrix A then identify an Eigen value of the matrix A^T

a) λ
b) $-\lambda$
c) $\frac{1}{\lambda}$
d) Can Not be determined

(viii) Examine the sum of the Eigen values of the matrix $A=\begin{pmatrix}1 & 1 & 3\\ 1 & 5 & 1\\ 3 & 1 & 1\end{pmatrix}$ is

a) 5
b) -5
c) 7
d) -7

(ix) If A is an skew-symmetric matrix then select which of the following be an possible Eigen value of A

a) 1
b) -1
c) 0
d) None of -1,0,1

(x) Eliminating arbitrary constants a and b from $z=(x^2+a)(y^2+b)$, construct the PDE

a) $p+q=xyz$
b) $pq=xyz$
c) $pq=4xyz$
d) $pq=-4x^2y^2z^2$

(xi) If is an orthogonal Matrix then identify the matrix A is

a) Singular Matrix
b) Non-Singular Matrix
c) Symmetric Matrix
d) Skew-Symmetric matrix

(xii) Let α, β, γ be three vectors in a vector space V over R, where R is the set of all real numbers. If $c\alpha+d\beta+e\gamma=\theta$, where θ is the zero vector in V then identify the value of c, d, e respectively.

a) 1,1,1
b) 0,0,0
c) 1,0,0
d) 0,1,1

(xiii) Construct the complete integral of the PDE $pq=z$, where $p=\frac{\partial z}{\partial x}$, $q=\frac{\partial z}{\partial y}$

a) $z=(x+a)(y+b)$
b) $z=x+y+a+b$
c) $z=\frac{x+a}{y+b}$
d) $z=xy+ax+by$

(xiv) Select the correct option: The value of $\begin{vmatrix}1 & \omega & \omega^2\\ \omega^2 & 1 & \omega\\ \omega^2 & \omega & 1\end{vmatrix}$ is .

a) 0
c) 2

b) 1
d) 3

(xv) Select the correct option: If the matrix $\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & \lambda \end{pmatrix}$ is singular then the value of λ is

a) 3
c) 2

b) 5
d) 4

Group-B
(Short Answer Type Questions)

3 x 5=15

2. A mapping $T : R^3 \to R^3$ is defined by $T(x_1, x_2, x_3) = (x_1 + x_2 + x_3, 2x_1 + x_2 + 2x_3, x_1 + 2x_2 + x_3), (x_1, x_2, x_3) \in R^3$. Show that T is a linear mapping. Find KerT and the dimension of the KerT. (3)

3. Examine that if λ is an Eigen value of an orthogonal matrix, and then show that $\frac{1}{\lambda}$ is also an Eigen value. (3)

4. Describe the characteristic equation and Eigen values of the matrix $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ (3)

5. Solve: $r + 2s + t = 2(y - x) + \sin(x - y)$ (3)

6. Solve $xdx + ydy + \frac{xdy - ydx}{x^2 + y^2} = 0$ (3)

OR

Solve the homogeneous differential equation $\frac{d^3y}{dx^3} - 4\frac{d^2y}{dx^2} + 5\frac{dy}{dx} - 2y = 0$ (3)

Group-C

(Long Answer Type Questions) 5 x 6=30

7. Show that every square matrix can be uniquely expressed as sum of symmetric and skew symmetric matrix. (5)

8. (5)

If $D = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}$ and its adjugate $D' = \begin{vmatrix} A & H & G \\ H & B & F \\ G & F & C \end{vmatrix}$, then develop that ,

$$\frac{CA-G^2}{b} = \frac{AB-H^2}{c} = D$$

9. (5)

Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, which satisfies the conditions

$u(0,y) = u(l,y) = u(x,0) = 0$ and $u(x,a) = \sin\frac{n\pi x}{l}$.

10. (5)

Recognize the eigenvalues of the matrix $A = \begin{pmatrix} 1 & -1 \\ 2 & -3 \end{pmatrix}$.

11. Use Gram-Schmidt process to identify an orthogonal basis from the basis set $\{(1,0,1), (1,1,1), (1,3,4)\}$ of the Euclidean space R^3 with standard inner product. (5)

12. (5)

Evaluate the values of a, b, c if the matrix $A = \begin{bmatrix} 0 & 2b & c \\ a & b & -c \\ a & -b & c \end{bmatrix}$ is orthogonal.

OR

If λ, η be two Eigen values of the matrix $\begin{bmatrix} 6 & 4 \\ -3 & -1 \end{bmatrix}$. Then evaluate λ, η and then evaluate the bases for the corresponding Eigen spaces. (5)

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