BRAINWARE UNIVERSITY

Term End Examination 2021 - 22
Programme – Master of Science in Mathematics
Course Name – Abstract Algebra
Course Code - MSCMC201
(Semester II)

Time allotted : 1 Hrs.15 Min. **Full Marks : 60**

[The figure in the margin indicates full marks.]

Group-A

(Multiple Choice Type Question) 1 x 60=60

Choose the correct alternative from the following :

(1) Let G be a simple group of order 168. How many subgroups of G are of order 7?

a) 1 b) 7

c) 8 d) 28

(2) A group of order 22 is

a) simple group b) is not a simple group

c) commutative group d) None of these

(3) A simple group of order 63

a) cannot contain a subgroup of order 21 b) can contain a subgroup of order 21

c) is commutative d) None of these

(4) The direct product of two groups is commutative if and only if

a) both the groups are commutative. b) both the groups are not commutative.

c) one of is commutative. d) None of these

(5) Let G be a finite group of order mn, where m and n are relatively prime. Let H and K be subgroups of G having orders m and n respectively, then $G=$

a) $K\times H$ b) $H\times K$

c) H d) None of these

(6) $Klein\ 4-group$ is a $p-group$, where $p=$

a) 3 b) 1

c) 5 d) 2

(7) Let G be a group of order p^2, where p is a prime. Then G

a) is commutative
b) is noncommutative
c) has no subgroup of order p
d) None of these

(8) Let G be a cyclic group of order mn, where m, n are positive integers such that $gcd(m,n)=1$, then G is

a) isomorphic to $Z_m \times Z_n$
b) not isomorphic to $Z_m \times Z_n$
c) isomorphic to Z_m
d) None of these

(9) The group of symmetrices of a square is a $p-group$. Then $p=$

a) 5
b) 2
c) 3
d) None of these

(10) Consider the subnormal series $Z \supset 6Z \supset 12Z \supset 48Z \supset \{0\}$, then the subnormal series $Z \supset 2Z \supset 6Z \supset 12Z \supset 48Z \supset \{0\}$ is

a) a refinement of the previous
b) not a refinement of the previous.
c) same as the previous.
d) None of these

(11) Every solvable series

a) is a composition series.
b) is not a composition series.
c) is a normal series.
d) None of these

(12) The symmetric group S_4

a) is commutative.
b) is not solvable.
c) is solvable.
d) None of these

(13) Let G be a simple and solvable group, then G is

a) not commutative.
b) cyclic.
c) commutative.
d) None of these

(14) Let p be a prime integer and $n>1$ be any integer and G be a group of order p^n, then

a) G is simple.
b) G is commutative.
c) G is not simple.
d) None of these

(15) Let G be a group of order 9, then

a) G is commutative.
b) G is non-commutative.
c) G is not simple.
d) None of these

(16) Let G be a group of order 10, then G

a) G is commutative.
b) G is non-commutative.
c) G is not simple.
d) None of these

(17) Let G be a cyclic group of order p^2, p is a prime, then

a) G has only one subgroup.
b) more than one subgroup.
c) G is not commutative.
d) None of these

(18) Let A and B be two cyclic groups of order m and n respectively. Then $A \times B$ is a cyclic group if and only if

a) $\gcd(m,n)=1$
b) $\gcd(m,n)=5$
c) $\gcd(m,n)=3$
d) None of these

(19) Any simple group of order 60 is isomorphic to

a) A_5
b) A_3
c) A_4
d) None of these

(20) Let G be a group of order 15, then G

a) has a unique Sylow 3-subgroup
b) has more than one Sylow 3-subgroup
c) has no Sylow 3-subgroup
d) has a Sylow 7-subgroup

(21) Let G be a group of order 36, then G

a) is not simple
b) is simple
c) is cyclic
d) None of these

(22) In the ring Z of integers, the invertible element is/are

a) only 1
b) only -1
c) both -1,1
d) only 0

(23) The ring Z_{12} is

a) an integral domain
b) a field
c) both integral domain and field
d) neither integral domain nor field

(24) The ring Z_n is a filed then n is always a/an

a) even prime
b) odd prime
c) prime
d) any integer

(25) The characteristic of the ring R of all real numbers is

a) 0
b) 1
c) - 1
d) does not exists

(26) The characteristic of the ring C of all complex numbers is

a) 0
b) 1
c) - 1
d) does not exists

(27) Let R be a ring with 1. Then R has characteristic n

a) $n.1=1$
b) $n.1=0$
c) $n.1=1, k<n \Rightarrow k.1\neq 1$
d) $n.1=0, k<n \Rightarrow k.1\neq 0$

(28) Which of the following is a zero of the polynomial $X^2+\bar{2}X+\bar{1}$ in the ring Z_4?

a) $\bar{0}$
b) $\bar{1}$
c) $\bar{2}$
d) No root

(29) Let R be ring with1. Then the $R[X]/\langle X\rangle =$

a) R
b) R[X]
c) R(X)
d) $\langle X\rangle$

(30) Let R[X] be a polynomial ring and $f(X), g(X)$ be two non-zero polynomials in R[X]. If $f(X)+g(X)\neq 0$ then $\max \deg(f(X), g(X))$

a) $=\deg(f(X))+\deg(g(X))$
b) $\geq \deg(f(X))+\deg(g(X))$
c) $\leq \deg(f(X))+\deg(g(X))$
d) None of these

(31) Which of the following factor divides the polynomial $2X^2+X+1$ in Z_3?

a) $X-1$
b) $X-2$
c) X
d) None of these

(32) Which of the following is not a unit in the ring Z[i]?

a) 0
b) 1
c) - 1
d) I

(33) Which of the following statements is true?

a) ED implies PID
b) PID implies ED
c) ED implies and implied by PID
d) None of these

(34) Which of the following factor divides the polynomial $2X^2+X+1$ in Z_5?

a) X-2
b) X-3
c) X
d) None of these

(35) The associates of 1+i in Z[i]

a) 1
b) i
c) 1+i
d) -i

(36) The associates of 1-i in Z[i]

a) 1
b) i
c) 1-i
d) -i

(37) Value of the *g.c.d*(10,15) in the ring Z

a) 1
b) 5
c) 10
d) 30

(38) Let R be a commutative ring with 1and A and B are two distinct maximal ideal of R then AB=

a) A+B
b) $A \cup B$
c) $A \cap B$
d) None of these

(39) The number of irreducible polynomial of degree two in the ring Z_2

a) 0
b) 1
c) 2
d) 3

(40) A rational root of the polynomial $2X^3-7x+1$

a) 1/2
b) 2/3
c) does not exists
d) exists but none of a and b

(41) Which of the following is an algebraic integer?

a) i
b) 1/2
c) $i/2$
d) None of these

(42) Let L/K be a finite extension of fields. Which of the following assertions are correct

a) If the characteristic of K is zero, then L/K is normal
b) If the characteristic of K is zero, then L/K is separable.
c) If L/K is normal, then L/K is a finite field extension
d) If the characteristic of K is positive, then L/K is normal if and only if it is separable.

(43) Which of the following is not an algebraic element over the set of all real numbers R?

a) π
b) i
c) $\sqrt{2}$
d) None of these

(44) The degree of i over the set of all real numbers R?

a) 0
b) 1
c) 2
d) 3

(45) Which of the following field is prime?

a) R
b) Q
c) C
d) $Q(\sqrt{2})$

(46) The degree of $\sqrt{2}$ over the set of all real numbers R

a) 0
b) 1
c) 2
d) 3

(47) The value of $[Q(\sqrt{2}):Q]$ is

a) 0
b) 1
c) 2
d) 3

(48) The value of $[Q(\sqrt{2},\sqrt{3}):Q(\sqrt{2})]$ is

a) 0
b) 1
c) 2
d) 3

(49) The basis of $Q(i)$ over Q is

a) $\{1,i\}$
b) $\{1\}$
c) $\{i\}$
d) None of these

(50) A field extension K(c)/K is finite then

a) c in K
b) c is algebraic over K
c) c is transcendental over K
d) None of these

(51) A field extension K(c)/K is finite only if

a) c in K
b) c is algebraic over K
c) c is transcendental over K
d) None of these

(52) The value of $[Q(\sqrt{2},\sqrt{3}):Q]$ is

a) 1
b) 2
c) 3
d) 4

(53) Which of the following element is transcendental over Q

a) π^2
b) $\sqrt{2}$
c) $\sqrt{-1}$
d) None of these

(54) Which of the following is the splitting field of the polynomial X^2+1 over Q

a) Q
b) $Q(i)$
c) C
d) R

(55) Which of the following is the splitting field of the polynomial X^2+1 over R

a) Q
b) $Q(i)$
c) C
d) R

(56) Let K be a perfect field then which of the following statements is true?

a) Every algebraic extension of K is separable
b) Some algebraic extension of K is separable
c) Every algebraic extension of K is inseparable
d) Some algebraic extension of K is inseparable

(57) Let K be a field of characteristic 5 and K is perfect if

a) $K = K^5$ b) $K \subset K^5$

c) $K \supset K^5$ d) None of these

(58) Which of the following is true?

a) $\mathbb{Q}(\sqrt{2}, i) = \mathbb{Q}(\sqrt{2} + i)$ b) $\mathbb{Q}(\sqrt{2}, i) \not\subset \mathbb{Q}(\sqrt{2} + i)$

c) $\mathbb{Q}(\sqrt{2}, i) \cap \mathbb{Q}(\sqrt{2} + i) = \mathbb{Q}$ d) None of these

(59) Multiplicity of the root 0 of the polynomial $X^3 + 2X = 0$

a) 3 b) 2

c) 1 d) 0

(60) Which of the following is not an irreducible polynomial with integer coefficients

a) $X^2 + X + 1$ b) $X^2 + 1$

c) $X + 1$ d) $X^2 + 2X + 1$