BRAINWARE UNIVERSITY

Term End Examination 2021 - 22
Programme – Master of Science in Mathematics
Course Name – Complex Analysis
Course Code - MSCMC202
(Semester II)

Time allotted : 1 Hrs.15 Min. **Full Marks : 60**

[The figure in the margin indicates full marks.]

Group-A

(Multiple Choice Type Question) 1 x 60=60

Choose the correct alternative from the following :

(1) The value of the integral $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-4)(z-2)} dz$, where C is the circle $|z| = 3$ and integration is taken anti-clockwise, is

a) $-2\pi i$ b) πi

c) $-\pi i$ d) $2\pi i$

(2) The value of the integral $\int_C \frac{dz}{z^2+9}$, where C is the ellipse $\frac{x^2}{4} + \frac{y^2}{1} = 1$ and the integral is taken in positive sense, is

a) 0 b) $2\pi i$

c) $-\pi i$ d) πi

(3) $f(z) = z^n (n \geq 2)$ is conformal

a) At every point in C b) At every point except 0 in C

c) At no point in C d) Only at $z = 0$

(4) $\int_C \frac{z^3 + 2z + 1}{(z+1)^3} dz$, where $C: |z| = 2$ is

a) 0 b) 1

c) $-6\pi i$ d) $4\pi i$

(5)

The value of $\int_C \frac{dz}{z+2}$, where $C: |z| = 1$ is

a) $\frac{\pi}{2}$

b) 1

c) $2\pi i$

d) 0

(6) A real valued function of a complex variable

a) Has derivative zero

b) Does not have derivative

c) Has derivative not necessarily zero

d) Either has derivative zero or the derivative do es not exist

(7) The function $f(z) = |z|$ is analytic

a) Everywhere

b) Nowhere

c) Only at $z = 0$

d) Everywhere except at $z = 0$

(8) If f is a bounded entire function then f is

a) Non-constant

b) Not differentiable

c) Not analytic

d) Constant

(9) Let f is continuous in an open connected set D and $\int_C f(z)dz = 0$ for each piecewise differentiable curve C in D. Then

a) f is differentiable on C

b) f is analytic on C

c) f is analytic in D

d) f is not necessarily analytic in D

(10) In complex form Cauchy-Riemann equation takes the form

a) $\frac{\partial f}{\partial x} = i\frac{\partial f}{\partial y}$

b) $\frac{\partial f}{\partial x} = \frac{1}{i}\frac{\partial f}{\partial y}$

c) $i\frac{\partial f}{\partial x} = -\frac{\partial f}{\partial y}$

d) $\frac{\partial f}{\partial x} = -\frac{1}{i}\frac{\partial f}{\partial y}$

(11) The function $f(z) = |z|^2$ is

a) Continuous nowhere

b) Continuous everywhere but no-where differen tiable

c) Continuous everywhere but no-where differen tiable except the origin

d) Continuous at origin only

(12) $\int_C \frac{1}{z} dz$, where $C: |z-2| = 1$ is

a) 0

b) $2\pi i$

c) 1

d) $2\pi i$

(13) $f(z) = \bar{z}$

a) No z

b) All z

c) z=0

d) z=1

(14) Which of the following functions is analytic?

a) $f(x+iy) = \exp(iy)$

b) $f(x+iy) = x$

c) $f(x+iy)=iy$

d) $f(x+iy)=x+iy$

(15) The integral $\int_{|z|=2} \frac{\cos z}{z^3}dz$, equals

a) πi

b) $-\pi i$

c) $2\pi i$

d) $-2\pi i$

(16) A given complex function of a complex variable takes real values for real values of the va riable. Then

a) It may be analytic in an open disc of the comp lex plane but not differentiable on any interval as a real function of a real variable

b) Its power series expansion, when available, ar ound a point on the real axis have only real co efficients

c) Its power series expansion, when available, ar ound any point in the complex plane has real c oefficients

d) It may not possess a power series expansion ar ound a point on the real axis as a real function but may be analytic in some open disc centred at the point

(17) Let $f(z)=\begin{cases}\frac{|z|}{\operatorname{Re} z}, \operatorname{Re} z \neq 0 \\ 0, \operatorname{Re} z = 0\end{cases}$ then $f(z)$

a) Has a non-zero limit as $z \to 0$

b) Is differentiable at z=0

c) Is continuous but not differentiable at z=0

d) Is neither continuous nor differentiable at z=0

(18) Let $E=\{z \in C : |z|\rangle 1\} \cup \{i\}$. then

a) E is open but not closed in C

b) E is closed but not open in C

c) E is neither open nor closed in C

d) E is both open and closed in C

(19) If $\left(\frac{1-i}{1+i}\right)^{100} = a+ib$, then

a) a = 2, b = -1

b) a = 1, b = 0

c) a = 0, b = 1

d) a = -1, b = 2

(20) Under the stereographic projection, the south pole goes to

a) (0,0)

b) (1,0)

c) (0,1)

d) (0,-1) in the complex plane

(21) Let f be an entire function such that $f(iy)=\exp(iy), 0 \le y \le 1$. then

a) $f(x+iy)=\exp(x+iy)$ for every x and y

b) $f(x+iy)=\overline{\exp(iy)}$

c) $f(x+iy)=\exp(x+iy)$ for every x and $0 \le y \le 1$.

d) None of the above

(22) Let $A=\{z \in C : |z-2|+|z+1| \ge 3\}$. then

a) A is bounded, closed subset of C

b) A is an unbounded proper subset of C

c) A=C

d) A is an unbounded subset of C which is not cl

(23) In the complex plane e^z assumes

a) Every positive value
b) Every negative value
c) Every value
d) Every value excepting zero

(24) The cross ratio (z_1, z_2, z_3, z_4) is real iff

a) The four points lie on a circle
b) The four points lie on a straight line
c) The four points lie on a circle or on a straight line according as none of z_1, z_2, z_3, z_4 is ∞ or one of z_1, z_2, z_3, z_4 is ∞
d) None of the above

(25) Let $f : C \to C$ be analytic such that $f\left(\frac{1}{n}\right) = \frac{1}{n^2}, n = 1, 2, \ldots$ then

a) f is bounded function
b) there does not exist such a function
c) $f(z) = z^2, \forall z \in C$
d) The whole complex plane C

(26) The equation $z\bar{z} + i\bar{z} - i\bar{z} - 3 = 0$ describes

a) A straight line
b) An ellipse
c) A circle
d) A pair of straight line

(27) $f(z) = \exp(z)$ is a periodic function of period

a) 2π
b) $2n\pi i$
c) $2\pi i$
d) π

(28) If $f(z) = u + iv$ and $u - v = e^x(\cos y - \sin y)$, then $f(z)$ in terms of z is

a) $\sin z + c$
b) $\cos z + c$
c) $e^z + c$
d) None of the above

(29) A Mobius transformation $f(z) \frac{az+b}{cz+d}$, $ad - bc \neq 0$, other than the identity transformation, has

a) No fixed point
b) Only one fixed point
c) At most two fixed points
d) Three or more than three fixed points

(30) The function $w(z) = -\left(\frac{1}{z} + bz\right), 1 < b < 1, maps\ |z| < 1$ onto

a) A half plane
b) Exterior of a circle
c) Exterior of an ellipse
d) Interior of an ellipse

(31) The transformation $w = e^{i\theta}\left(\frac{z-p}{\bar{p}.(z-1)}\right)$, where p is a constant, maps $|z| < 1$ onto

a) $|w| < 1$ if $|p| < 1$
b) $|w| > 1$ if $|p| > 1$
c) $|w| = 1$ if $|p| = 1$
d) $|w| = 3$ if $p = 0$

(32)

The set $\{z \in C: |z-2| + |z-1| < 3\}$ describes

a) The interior of a disc

b) The interior of an ellipse

c) The null set

d) The whole complex plane

(33) The equation $z\bar{z} + i\bar{z} - 3$ describes

a) A straight line

b) An ellipse

c) A circle

d) A pair of straight line

(34) For any closed curve γ, $a \in \gamma$, the index (or, winding) number $\eta(\gamma; a)$ is

a) an integer

b) a rational number

c) an irrational number

d) a fraction

(35) The value of the integral $\int_{|z|=1} \frac{\sin z}{z} dz$ is

a) 0

b) 1

c) $2\pi i$

d) 2π

(36) Let $f(z) = \frac{e^z}{z^3}$ if $z \neq 0$. The residue of at z=0

a) 0

b) 1

c) $2\pi i$

d) ½

(37) Let f be meromorphic function. Then its

a) Zeroes are isolated points but poles are not so

b) Poles are isolated but zeroes are not so

c) Both poles and zeroes are isolated points

d) Neither zeroes nor poles are isolated

(38) Under Mobious transformation cross-ratio is

a) Variable

b) Invariant

c) Zero

d) None of these

(39) A rational function $f(z) = \frac{az+b}{cz+d}$ $a, b, c, d \in C$ and $ad - bc \neq 0$, is known as Mobius function or bilinear function. It is analytic

a) Everywhere

b) Nowhere

c) Everywhere except at $z = -\frac{d}{c}$

d) Only at $z = -\frac{b}{a}$

(40) Which of the following function $f(z)$ satisfies Cauchy- Riemann equations?

a) $f(z)=\bar{z}$ at $z=1+i$

b) $f(z)=|z|^2$

c) $f(z)=\sqrt{|xy|}$

d) $f(z)=\frac{x^3(1+i)-y^3(1-i)}{x^2+y^2}, z\neq 0, f(0)$

(41) The analytic function $w=u+iv$ where $u=e^{-x}\{(x^2-y^2)\cos y+2xy\sin y\}$ is

a) $e^{-x}\{(x-iy)^2(\cos y-i\sin y\}$

b) $e^{-x}\{(x-iy)^2(\cos y+i\sin y\}$

c) $e^{-x}\{(x+iy)^2(\cos y-i\sin y\}$

d) $e^{-x}\{(x-iy)^2(\cos y+i\sin y\}$

(42) Which of the following is not correct for analytic functions $f(z)$ and $g(z)$ in a region G ?

a) $f(z)$ - $g(z)$ is analytic in G

b) $f(z)\ g(z)$ is analytic in G

c) $f(z)/g(z)$ is analytic in G

d) $f(z)+g(z)$ is analytic in G

(43) The Mobius transformation which maps the points $z_1=-1, z_2=0, z_3=1$ onto the points $w_1=-i, w_2=1, w_3=i$ respectively is

a) $w=Re(z)$

b) $w=Im(z)$

c) $w=\frac{z-i}{z+i}$

d) $w=\bar{z}$

(44) The function $f(z)=e^x(\cos ky=i\sin ky), z=x+iy$, is analytic iff $k=$?

a) 1

b) 0

c) 2

d) none of these

(45) The set of all Bilinear transformation form an algebraic structure under the composition of produ ct of transformations this algebraic structure is

a) An algebraic group

b) A cycle group

c) A non-abelian group

d) A ring

(46) Which of the following functions $f(z)$ is analytic and bounded where $f(z)=$

a) $\sin z$

b). $\cos z$

c) Any polynomial of degree more than one

d) None of these

(47) Every bilinear transformation maps a circle or straight line into

a) Straight line or circles respectively

b) Circle or straight line respectively

c) Circles

d) Straight line

(48) The transformation $f(z)=x-iy$ is:

a) Conformal

b) Isogonal

c) Analytic

d) None of these

(49) Under the transformation $w=z+1-i$, the image of the line $x=0$ in the $z-plane$ is:

a) $u=1$

$v=1$

b) $u=0$

c)

d) $v=0$

(50) A transformation of the type $w=\alpha z+\beta$, where α and β are complex constants, is known as a:

a) Translation

b) Magnification

c) Linear transformation

d) Bilinear transformation

(51) A bilinear transformation $w=\frac{az+b}{cz+d}$ having only one fixed point is called

a) Parabolic transformation

b) Elliptic transformation

c) Hyperbolic transformation

d) None of these

(52) Which of the following is a bilinear transformation :

a) $w=\dfrac{2z+1}{4z+2}$

b) $w=\dfrac{(2+3i)z+i}{-13iz+(2-3i)}$

c) $w=z^2$

d) $w=\dfrac{(1+i)z+1}{2z+(1-i)}$

(53) A rational function of the form $f(z)=\frac{az+b}{cz+d}$, $a,b,c,d\in C$ $ad-bc\neq 0$ is known as Mobius function or bilinear function. It is analytic

a) Everywhere

b) Nowhere

c) Everywhere expect at $z=\frac{-b}{c}$

d) None of these

(54) Under the transformation $w=\dfrac{1}{z}$, the image of the line $y=\dfrac{1}{4}$ in the $z-plane$ is:

a) Circle $u^2+v^2+4v=0$

b) Circle $u^2+v^2=4$

c) Straight line

d) None of them

(55) If a function is analytic at all points of a bounded domain except at finitely many points, then these points are called:

a) Zeros

b) Singularities

c) Poles

d) Simple points

(56) for the function $f(z)=\tan\left(\dfrac{1}{z}\right)$, $z=0$ is:

a) isolated essential singularity

b) non-isolated essential singularity

c) pole

d)

none of these

(57) for the function $f(z)=e^{z}, z=\infty$

a) isolated essential singularity

b) pole

c) ordinary point

d) none of these

(58) A mobius transformation $f(z)=\frac{az+b}{cz+d}, a, b, c, d \in C\ ad-bc \neq 0$, other than the identity transformation, has

a) No fixed point

b) Only one fixed point

c) At most two fixed point

d) Three or more than three fixed point

(59) $z=0$ for the function $f(z)=\log z$ is:

a) Isolated singularity

b) Pole

c) Non-isolated singularity

d) None of these

(60) If $f(z)=u+iv$ and $u-v=e^{x}(cosy-siny), then\ f(z)$ in terms of z is

a) Sin z+c

b) Cos z+c

c) e^{x}+c

d) None of these