BRAINWARE UNIVERSITY

Term End Examination 2021 - 22
Programme – Master of Science in Mathematics
Course Name – Partial Differential Equations
Course Code - MSCMC203
(Semester II)

Time allotted : 1 Hrs.15 Min. **Full Marks : 60**

[The figure in the margin indicates full marks.]

Group-A

(Multiple Choice Type Question) 1 x 60=60

Choose the correct alternative from the following :

(1) The general solution of $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$ is of the form

a) $u=f(x+iy)+g(x-iy)$
b) $u=f(x+y)+g(x-y)$
c) $u=cf(x-iy)$
d) $u=g(x+y)$

(2) The initial value problem $u_x+u_y=1,\ u(s,s)=\sin s,\ 0\le s\le 1$ has

a) two solutions
b) a unique solution
c) no solution
d) infinitely many solutions

(3) In the region $x>0, y>0$, the partial differential equation $(x^2-y^2)\frac{\partial^2 u}{\partial x^2}+2(x^2+y^2)\frac{\partial^2 u}{\partial x\partial y}+(x^2-y^2)\frac{\partial^2 u}{\partial y^2}=0$

a) changes type
b) is elliptic
c) is parabolic
d) is hyperbolic

(4) The governing equations of CFD are _____________ partial differential equations.

a) Linear
b) Quasi-linear
c) Non-linear
d) Non-homogeneous

(5) The variables ξ and η which reduce the differential equation $\frac{\partial^2 u}{\partial x^2}-x^2\frac{\partial^2 u}{\partial y^2}=0$ to the canonical form, are

a) $\xi=y^2+\frac{1}{2}x,\ \eta=y^2-\frac{1}{2}x$
b) $\xi=y+\frac{1}{2}x^2,\ \eta=y-\frac{1}{2}x^2$

c) $\xi = y + x^2,\ \eta = y - x^2$

d) $\xi = y^2 + x,\ \eta = y^2 - x$

(6) The solution of the differential equation $r + 5s + 6t = (y - 2x)^{-1}$ is

a) $\phi_1(y+2x) + \phi_2(y+3x) + \log(y+2x)$

b) $\phi_1(y-2x) + \phi_2(y+3x) + x\log(y-2x)$

c) $\phi_1(y-2x) + \phi_2(y-3x) + x\log(y+2x)$

d) $\phi_1(y-2x) + \phi_2(y-3x) + x\log(y-2x)$

(7) The singular solution of the differential equation $(xp - y^2) = p^2 - 1$ is

a) $x^2 + y^2 = 1$

b) $y^2 - x^2 = 1$

c) $x^2 + 2y^2 = 1$

d) $x^2 - y^2 = 1$

(8) Which of these is a quasi-linear partial differential equation?

a) $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = 0$

b) $\dfrac{\partial^2 u}{\partial x^2} + a(x,y)\dfrac{\partial^2 u}{\partial y^2} = 0$

c) $\dfrac{\partial u}{\partial x}\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial u}{\partial y}\dfrac{\partial^2 u}{\partial y^2} = 0$

d) $\left(\dfrac{\partial^2 u}{\partial x^2}\right)^2 + \dfrac{\partial^2 u}{\partial y^2} = 0$

(9) The solution of the given differential equation $\dfrac{\partial^4 z}{\partial x^4} - \dfrac{\partial^4 z}{\partial y^4} = 0$, is

a) $f_1(y+x) + f_2(y-x) + f_3(y+ix) + f_4(y-i$

b) $f_1(y+x) + f_2(y-x)$

c) $f_1(y+ix) + f_2(y-ix)$

d) None of these

(10) A partial differential equation has

a) one independent variable

b) two or more independent variables

c) more than one dependent variable

d) equal number of dependent and independent variables

(11) Classify the heat equation $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} + \dfrac{\partial^2 u}{\partial z^2} = \dfrac{1}{c^2}\cdot\dfrac{\partial u}{\partial t}$

a) Elliptic

b) Hyperbolic

c) Parabolic

d) None of these

(12) Classify the equation $\dfrac{\partial^2 u}{\partial x^2} - \dfrac{\partial^2 u}{\partial x \partial t} + 4\dfrac{\partial^2 u}{\partial x^2} = 0$

a) Elliptic

b) Hyperbolic

c) Parabolic

d) None of these

(13) The following is true for the following partial differential equation used in nonlinear mechanics known as the Korteweg-de Vries equation.

$$\frac{\partial v}{\partial t} + \frac{\partial^3 v}{\partial x^3} - 6v\frac{\partial v}{\partial x} = 0$$

a) linear; 3rd order

b) nonlinear; 3rd order

c) linear; 1st order

d) nonlinear; 1st order

(14) The equations $R\,dp\,dy + T\,dq\,dx - V\,dx\,dy = 0$ and $R\,dy^2 - S\,dx\,dy + T\,dx^2 = 0$ are called

a) Lagrange's Auxiliary Equations
b) Lagrange's subsidiary equations
c) Monge's subsidiary equations
d) Monge's auxiliary equations

(15) One dimensional wave equation is given by

a) $\frac{\partial^2 u}{\partial t^2} = C^2 \frac{\partial^2 u}{\partial x^2}$

b) $\frac{\partial^2 u}{\partial t^2} = \frac{1}{C^2} \frac{\partial^2 u}{\partial x^2}$

c) $\frac{\partial^2 u}{\partial t^2} + C^2 \frac{\partial^2 u}{\partial x^2} = 0$

d) $\frac{\partial^2 u}{\partial t^2} + \frac{1}{C^2} \frac{\partial^2 u}{\partial x^2} = 0$

(16) The solution of $\frac{\partial^3 z}{\partial x^3} = 0$, is

a) $z = \left(1 + x + x^2\right) f(y)$

b) $z = \left(1 + y + y^2\right) f(x)$

c) $z = f_1(y) + x f_2(y) + x^2 f_3(y)$

d) $z = f_1(x) + y f_2(x) + y^2 f_3(x)$

(17) The solution of $\frac{\partial^2 z}{\partial x^2} - 5\frac{\partial^2 z}{\partial x \partial y} + 4\frac{\partial^2 z}{\partial y^2} = \sin(4x + y)$ is

a) $z = \frac{1}{3} x \cos(4x + y)$

b) $z = f_1(y + x) + f_2(y + 4x)$

c) $z = f(y + x) - \frac{1}{3} x \cos(4x + y)$

d) $z = f_1(y + x) + f_2(y + 4x) - \frac{1}{3} x \cos(4x + 3y)$

(18) The relation $z = (x + a)(y + b)$ represents the PDE

a) $z = \frac{p}{q}$

b) $z = pq$

c) $z = p - q$

d) None of these

(19) The complete solution of $z = px + qy + p^2 + q^2$ is

a) $z = ax + by + a^2 + b^2$

b) $z = ax + by$

c) $z = a^2 x^2 + b^2 y^2$

d) None of these

(20) A surface passing through the two lines $z = x = 0$, $z - 1 = x - y = 0$, satisfying $r - 4s + 4t = 0$, is

a) $z = \frac{2x}{3x + y}$

b) $z = \frac{3x}{2x + y}$

c) $z = \frac{x + y}{3x - y}$

d) $z = \frac{2x}{3x + 2y}$

(21) The equation $x^2(y-1)z_{xx} - x\left(y^2 - 1\right)z_{xy} + y(y-1)z_{yy} + z_x = 0$ is hyperbolic in the entire xy-plane except along

a) x-axis
b) y-axis
c) a line parallel to y-axis
d) none of these

(22) The singular solution of $z^2\left(p^2 + q^2 + 1\right) = c^2$

a) does not exist

b) is $z = \pm c$

c) is $z = 0$

d) is $z = x + y$

(23) For the PDE $\frac{\partial z}{\partial x} + 2xy^3 \frac{\partial z}{\partial y} = z^3$, the general solution can be expressed in the form $F(u, v) = 0$ where u and v are

a) $u(x,y,z) = x^2 + y^{-2}$
$v(x,y,z) = x - \frac{1}{2} z^{-2}$

b) $u(x,y,z) = x^2 - y^2$
$v(x,y,z) = x - z^{-2}$

c) $u(x,y,z) = x^2 - \frac{1}{2} y^2$
$v(x,y,z) = x - \frac{1}{2} z^{-2}$

d) $u(x,y,z) = x^2 + \frac{1}{2} y^{-2}$
$v(x,y,z) = x + \frac{1}{2} z^{-2}$

(24) Let $u = f(x+iy) + g(x+iy)$, where f and g are arbitrary functions differentiable any order. Then the partial differential equation of minimum order satisfied by u is

a) $u_{xx} + 2u_{xy} + u_{yy} = 0$

b) $u_{xx} + u_{yy} = 0$

c) $\frac{u_{xx}}{x} + \frac{u_{yy}}{y} = 0$

d) $yu_{xx} + xu_{yy} = 0$

(25) Laplace's equation is

a) $u_{xx} + u_{yy} - u_z = 0$

b) $u_{xx} + u_{yy} + u_z^{\ 2} = 0$

c) $u_{xx} + u_{yy} - u_{zz} = 0$

d) $u_{xx} + u_{yy} + u_{zz} = 0$

(26) Suppose $u(x, y)$ satisfies Laplace's equation $\nabla^2 u = 0$ in $\mathbb{R}^2$ and $u = x$ on the unit circle. Then at the origin

a) u tends to infinity.

b) u attains a finite minimum.

c) u attains a finite maximum.

d) u is equal to 0.

(27) Consider the boundary value problem: $u_{xx} + u_{yy} = 0$ in $\Omega = \{(x, y) : x^2 + y^2 < 1\}$ with $\frac{\partial u}{\partial n} = x^2 + y^2$ on the boundary of Ω ($\frac{\partial u}{\partial n}$ denotes the normal derivative of u). Then its solution $u(x,y)$

a) is unique and is identically zero.

b) is unique up to a constant.

c) does not exist.

d) is unique and non-zero.

(28) Which of the following is elliptic?

a) Laplace equation

b) Wave equation

c) Heat equation

d) All of these

(29) The solution of Laplace equation in spherical polar coordinates when it is axially sy mmetric about Z-axis involves

a) Associated Legendre's function

b) Legendre's polynomial

c) Bessel's function

d) Trigonometric function

(30) Which of the following concerning the solution of the Dirichlet problem for a smoo th bounded region is true?

a) Solution is unique

b) Solution is unique upto an additive consta nt

c) Solution is unique up to a multiplicative c onstant.

d) No conclusion can be made about uniquen ess.

(31) Which of the following is elliptic?

a) Laplace equation

b) Wave equation

c) Heat equation

d) $u_{xx} + 2u_{xy} - 4u_{yy} = 0$

(32) Consider the BVP $u_{xx} + u_{yy} = 0$, $x \in (0, \pi)$, $y \in (0, \pi)$, $u(x,0) = u(x,\pi) = u(0,y) = 0$.
Any solution of this BVP is of the form

a) $\sum_{n=1}^{\infty} a_n \sinh nx \sin ny$

b) $\sum_{n=1}^{\infty} a_n \cosh nx \sin ny$

c) $\sum_{n=1}^{\infty} a_n \sinh nx \cos ny$

d) $\sum_{n=1}^{\infty} a_n \cosh nx \cos ny$

(33) Poisson's equation is given as

a) $E = -\nabla^2 V$

b) $\nabla^2 V = -\frac{\rho v}{\varepsilon}$

c) $\nabla^2 V = 0$

d) All of these

(34) Solution of $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$ by method of separation of variables, given $z\left(x, \frac{\pi}{2}\right) = 0$, $z(x,0) = 4e^{-3x}$

a) $3e^{-4x} \cos 4y$

b) $4e^{-3x} \cos 3y$

c) $3e^{-3x} \cos 4y$

d) $4e^{-4x} \cos 3y$

(35) D'Alembert's solution of the wave equation is

a) $u(x,t) = \phi(x+ct) - \psi(x-ct)$

b) $u(x,t) = \phi(x+ct) + \psi(x-ct)$

c) $u(x,t) = \phi'(x+ct) - \psi'(x-ct)$

d) $u(x,t) = \phi(x-ct) + \psi(x+ct)$

(36) Solution of one dimensional wave equation is

a) $u_n(x,t) = \left(C_n \cos\frac{n\pi ct}{l} + D_n \sin\frac{n\pi ct}{l}\right)\sin\frac{n\pi}{l}$

b) $u_n(x,t) = \left(C_n \cos^2\frac{n\pi ct}{l} + D_n \sin^2\frac{n\pi ct}{l}\right)\cos$

c) $u_n(x,t) = \left(C_n \cos\frac{n\pi ct}{l} - D_n \sin\frac{n\pi ct}{l}\right)\sin\frac{n\pi}{l}$

d) None of these

(37) Solution of $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$, where $u(0,t) = 0$, $u(1,t) = 0$, $u(x,0) = A \sin \pi x$, $\left(\frac{\partial u}{\partial t}\right)_{t=0} = 0$, is

a) $u(x,t) = A\sin 2\pi ct \cos \pi x$

b) $u(x,t) = A\cos \pi ct \sin \pi x$

c) $u(x,t) = A\sin 2\pi ct \sin \pi x$

d) None of these

(38) Two dimensional wave equation is

a) $\frac{\partial^2 u}{\partial t^2} = c^2\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$

b) $\frac{\partial^2 u}{\partial x^2} = c^2\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$

c) $\frac{\partial^2 u}{\partial z^2} = c^2\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$

d) None of these

(39) Value of the constant C_n in solution of one dimensional wave equation is

a) $\frac{1}{l}\int_0^l f(x)\sin\frac{n\pi x}{l}dx$

b) $\frac{2}{l}\int_0^l f(x)\sin\frac{n\pi x}{l}dx$

c) $\frac{1}{l}\int_0^l f'(x)\cos\frac{n\pi x}{l}dx$

d) None of these

(40) Model of vibrating elastic string consists of

a) one dimensional wave equation

b) two dimensional wave equation

c) three dimensional wave equation

d) four dimensional wave equation

(41) Let $u(x,y)$ be a solution of the IVP $\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = 0$, $u(x,0) = x^2$, $u_t(x,0) = 0$ then $u(0,1) =$

a) 1

b) 0

c) 2

d) 1/2

(42) A function $u(x,t)$ satisfies the wave equation $\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = 0\ (0 < x < 1,\ t > 0)$. If $u\left(\frac{1}{2},0\right) = \frac{1}{4}, u\left(1,\frac{1}{2}\right) = 1$ and $u\left(0,\frac{1}{2}\right) = \frac{1}{2}$ then $u\left(\frac{1}{2},1\right) =$

a) $\frac{7}{4}$

b) $\frac{5}{4}$

c) $\frac{4}{5}$

d) $\frac{4}{7}$

(43) When solving a 1-Dimensional wave equation using variable separable method, we get the solution if

a) k is positive

b) k is negative

c) k is 0

d) k can be anything

(44) Solution of $\frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 u}{\partial x^2}$, $u(0,t) = u(l,t) = 0$, $u(x,0) = A\sin \pi x$ and $u_t(x,0) = 0$ is

a) $u = A\cos(\pi ct)\sin(\pi x)$

b) $u = A\sin(\pi ct)\cos(\pi x)$

c) $u = A\sin(\pi ct)\sin(\pi x)$

d) $u = A\cos(\pi ct)\cos(\pi x)$

(45) Solution of $u_{tt} = 4u_{xx}$ where $u(0,t) = u(5,t) = 0$, $u(x,0) = 0$ and $u_t(x,0) = 5\sin\pi x$ is

a) $\frac{5}{\pi}\sin 2\pi x \sin \pi t$

b) $\frac{5}{\pi}\cos \pi x \cos 2\pi t$

c) $\frac{5}{\pi}\cos 2\pi x \cos \pi t$

d) $\frac{5}{\pi}\sin \pi x \sin 2\pi t$

(46) Solution of $pt - qs = q^3$ is

a) $y = xz + f(z) + g(z)$

b) $y = xz + f(x) + g(z)$

c) $y = xz + f(x) + g(x)$

d) None of these

(47) Number of arbitrary constants in singular solution of an equation of degree n are

a) n

b) n-1

c) 0

d) 1

(48) Monge's method is used to solve the PDE of

a) n^{th} order

b) 2^{nd} order

c) 1^{st} order

d) Linear equation

(49) The order of the equation $r - t = x - y$ is

a) 1

b) 2

c) 3

d) 0

(50) The equation of the envelope of surface represented by complete integral of the giv en PDE is called

a) Singular solution

b) Particular Integral

c) General integral

d) None of these

(51) Monge's subsidiary equation for $s^2 = a^2 t$ are

a) $(dy)^2 + a^2(dx)^2 = 0$ and $dpdy - a^2 dxdq = 0$

b) $(dy)^2 - a^2(dx)^2 = 0$ and $dpdy - a^2 dxdq = 0$

c) $(dy)^2 - a^2(dx)^2 = 0$ and $dpdy + a^2 dxdq = 0$

d) $(dy)^2 + a^2(dx)^2 = 0$ and $dpdy + a^2 dxdq = 0$

(52) For the equation $z = pq$, Charpit's auxiliary equations are

a) $\frac{dp}{p} = \frac{dq}{q} = \frac{dz}{2pq} = \frac{dx}{q} = \frac{dy}{p}$

b) $\frac{dp}{q} = \frac{dq}{p} = \frac{dz}{pq} = \frac{dx}{p} = \frac{dy}{q}$

c) $\frac{dp}{p} = \frac{dq}{q} = \frac{dz}{pq} = \frac{dx}{p} = \frac{dy}{q}$

d) None of these

(53) Lagrange's subsidiary equations for $y^2zp + zx^2q = xy^2$ are

a) $\frac{dx}{y^2z} = \frac{dy}{zx^2} = \frac{dz}{y^2}$

b) $\frac{dx}{x^2} = \frac{dy}{y^2} = \frac{dz}{zx}$

c) $\frac{dx}{1/x^2} = \frac{dy}{1/y^2} = \frac{dz}{1/zx}$

d) None of these

(54) The solution of $xu_x + yu_y = 0$ is of the form

a) $f\left(\frac{y}{x}\right)$

b) $f(y+x)$

c) $f(x-y)$

d) $f(xy)$

(55) If the partial differential $(x-1)^2 u_{xx} - (y-2)^2 u_{yy} + 2xu_x + 2yu_y + 2xyu = 0$ is parabolic in $S \subseteq \mathbb{R}^2$ but not in $\mathbb{R}^2 / S$, then S is

a) $\{(x,y) \in \mathbb{R}^2 : x = 0 \text{ or } y = 2\}$

b) $\{(x,y) \in \mathbb{R}^2 : x = 1 \text{ or } y = 2\}$

c) $\{(x,y) \in \mathbb{R}^2 : x = 1\}$

d) $\{(x,y) \in \mathbb{R}^2 : y = 2\}$

(56) Heat equation is

a) $\frac{\partial u}{\partial x} = c^2 \nabla^2 x^2$

b) $\frac{\partial u}{\partial y} = \frac{1}{c^2} \nabla^2 t^2$

c) $\frac{\partial u}{\partial t} = c^2 \nabla^2 u$

d) $\frac{\partial u}{\partial t} = c^2 \nabla^2 y^2$

(57) In two dimension heat flow, the temperature along the normal to the *xy*-plane is

a) zero

b) infinity

c) finite

d) 100K

(58) While solving a partial differential equation using a variable separable method, we equate the ratio to a constant which?

a) can be positive or negative integer or zero

b) can be positive or negative rational numbe r or zero

c) must be a positive integer

d) must be a negative integer

(59) When solving a 1-Dimensional heat equation using a variable separable method, w e get the solution if

a) k is positive

b) k is negative

c) k is 0

d) k can be anything

(60) The solution of $\frac{\partial u}{\partial x} = 36\frac{\partial u}{\partial t} + 10u$ if $\frac{\partial u}{\partial x}(t=0) = 3e^{-2x}$ using the method of separation of variables, is

a) $-\frac{3}{2}e^{-2x}e^{-t/3}$

b) $3e^{x}e^{-t/3}$

c) $\frac{3}{2}e^{2x}e^{-t/3}$

d) $3e^{-x}e^{-t/3}$