BRAINWARE UNIVERSITY

Term End Examination 2021 - 22
Programme – Master of Science in Mathematics
Course Name – Mathematical Modelling
Course Code - MSCME402
(Semester IV)

Time allotted : 1 Hrs.15 Min. **Full Marks : 60**

[The figure in the margin indicates full marks.]

Group-A

(Multiple Choice Type Question) 1 x 60=60

Choose the correct alternative from the following :

(1) In population growth model, if a>0 (i.e., birth rate – death rate >0), then the population will become double its present size at time

a) $\frac{1}{a}\ln\frac{1}{2}$ b) $\frac{1}{a}\ln 2$

c) $\frac{1}{2}\ln\frac{1}{a}$ d) $\frac{1}{2}\ln a$

(2) Let x (t) is the population size at time t. If the birth rate is equal to the death rate, then the population size

a) grows exponentially b) decays exponentially

c) remains constant d) None of these

(3) In xy-plane, the curve passing through (0,3) point and having the tangent of slope $2x/y^2$ at any point (x, y) is

a) $\frac{1}{3}y^3 = x^2 + C$ b) $\frac{1}{9}y^3 = x^2 + 9$

c) $\frac{1}{3}y^3 = x^2 + 9$ d) None of these

(4) Suppose that a quantity y = y(t) has an exponential growth model with growth constant k > 0. Then y(t) satisfies a first-order differential equation of the form

a) $\frac{dy}{dt} = ky$ b) $\frac{dy}{dt} = -ky$

c) $\frac{dy}{dt} = \frac{k}{y}$ d) None of these

(5) Suppose that the half-life of a radioactive element is 1 minute. If 32 g of the element are available in a container at 1:00 p.m., then the amount remaining at 1:05 p.m. will be

a) 6.4 g
b) 4 g
c) 2 g
d) 1 g

(6) Suppose that a square is growing in such a way that the rate of change of its area is equal in magnitude to its perimeter. If A = A(t) is the area of the square at time t, then A(t) satisfies the first-order equation

a) $\frac{dA}{dt} = A$
b) $\frac{dA}{dt} = 4A$
c) $\frac{dA}{dt} = 4\sqrt{A}$
d) $\frac{dA}{dt} = 2\sqrt{A}$

(7) At time t = 0, a tank contains 30 oz of salt dissolved in 60 gal of water. Then brine containing 5 oz of salt per gallon of brine is allowed to enter the tank at a rate of 3 gal/min and the mixed solution is drained from the tank at the same rate. Give an initial-value problem satisfied by the amount of salt y(t) in the tank at time t .

a) $\frac{dy}{dt} + \frac{y}{20} = 30,\ y(0) = 15$
b) $\frac{dy}{dt} + \frac{y}{20} = 15,\ y(0) = 30$
c) $\frac{dy}{dt} + \frac{y}{30} = 20,\ y(0) = 15$
d) $\frac{dy}{dt} + \frac{y}{20} = 30,\ y(0) = 15$

(8) Scientists used mathematical models to predict the growth of world population, computer converted that data into a

a) sample
b) model
c) design
d) structure

(9) What is a Mathematical Model?

a) The creation of mathematical formulas to represent a real world problem in mathematical terms.
b) The creation of real world problems based solely on a theoretical formula already in existence.
c) Mathematical Models are toys that can be purchased in hobby shops by mathematicians.
d) None of these

(10) Regarding the technique of mathematical modeling, which of the following is the correct order, if

P: Mathematical problem

Q: Real problem

R: Interpretation

S: Mathematical Solution

a) PQRSP
b) QPRSQ
c) SPRQS
d) QPSRQ

(11) Out of all rectangles with a given perimeter, the _____ has the maximum area.

a) rhombus
b) circle
c) square
d) parallelogram

(12) If the sum of n positive numbers is constant, then their product is maximum when the numbers are

a) positive
b) negative
c) equal
d) None of these

(13) The particles in a molecular model follow

a) Discrete model
b) Continuous model
c) Linear model
d) None of these

(14) The model in which every set of variable states is uniquely determined by parameters in the model and by sets of previous states of these variables is termed as

a) Deterministic model
b) Probabilistic model
c) Statistic model
d) Stochastic model

(15) Model rests on neither theory nor observation, but is merely the invocation of expected structure is

a) Deductive model
b) Inductive model
c) Floating model
d) Constant model

(16) The model in which same set of parameter values and initial conditions will lead to an ensemble of different outputs, is

a) Deterministic model
b) Probabilistic model
c) Statistic model
d) Stochastic model

(17) To solve engineering problem, we have to formulate the pattern as math expression in term of variables, functions and equations, such expression is called

a) function model
b) math model
c) variable model
d) None of these

(18) According to Newton's law of cooling "The change of temperature of a body is proportional to the difference between the temperature of a body and that of the surrounding medium". If T1℃ is the initial temperature of the body and T2℃ is the constant temperature of the medium, T℃ be the temperature of the body at any time t then find the expression for T℃ as a function of T1℃, T2℃ and time t.

a) T=T1+(T2)exp(-kt)
b) T=T2+(T1-T2)exp(-kt)
c) T2+(T1-T2)exp(kt)
d) T=T2+(T1)exp(-kt)

(19) A bottle of mineral water at a room temperature of 72°F is kept in a refrigerator where the temperature is 44°F.After half an hour water cooled to 61°F.What is the temperature of the body in another half an hour?(Take $log\ \frac{28}{17} = 0.498$, $e^{-0.99} = 0.37$)

a) 18°F
b) 9.4°F
c) 54.4°F
d) 36.4°F

(20) How many milligrams of tritium will remain after 49.2 years if the starting amount is 32 mg? The half-life of tritium is 12.3 years.

a) 8 mg
b) 2 mg
c) 1 mg
d) 4 mg

(21) What would be the remaining concentration of 300 g of radioactive substance after 18 hours if the half-life is 3 hours?

a) 9.37 g
b) 2.34 g
c) 3.34 g
d) 4.68 g

(22) Suppose n chemical substances combine in the ratio $a_1 : a_2 : \ldots : a_n$ to form a third substance Z and z(t) is the amount of the third substance at time t. If $A_1, A_2, \ldots, A_n$ are the initial amounts of those n substances, then we get the non-linear equation

$$\frac{dz}{dt} = k(A_1 - a_1 z)(A_2 - a_2 z)\ldots(A_n - a_n z)$$

where $a_1 + a_2 + \ldots + a_n =$

a) 0
b) 1
c) $A_1 + A_2 + \ldots + A_n$
d) None of these

(23) The velocity of a particle (v) moving with simple harmonic motion, at any instant is given by

a) $\omega\sqrt{r^2 - x^2}$
b) $\omega\sqrt{x^2 - r^2}$
c) $\omega^2\sqrt{r^2 - x^2}$
d) $\omega^2\sqrt{x^2 - r^2}$

(24) Two balls of different masses (one lighter and one heavier) are thrown vertically upward with same initial speed. Which one will rise to a greater height?

a) The lighter one
b) The heavier one
c) Both the balls
d) None of these

(25) If $y_1(x)$ and $y_2(x)$ are linearly independent solutions to a second-order linear homogeneous differential equation (with continuous coefficient functions), then the general solution to this differential equation is

a) $y = c_1 y_1 + c_2 y_2$
b) $y = c_1 y_1 + c_2 x y_2$
c) $y = c_1 e^{y_1} + c_2 e^{y_2}$
d) None of these

(26) Which of the functions f and g are linearly independent?

a) $f(x) = \sin x \cos x,\ g(x) = \sin 2x$
b) $f(x) = \ln x,\ g(x) = \ln\sqrt{x}$
c) $f(x) = e^x,\ g(x) = e^{2x}$
d) $f(x) = \sin^2 x,\ g(x) = 1 - \cos 2x$

(27) Consider a disease 'X'. People who are diagnosed in the earlier stage have high chance of recovery. But the intense infection of 'X' will lead to death. The recovered people also stand a chance to get infected again. What kind of model does this disease 'X' exhibit?

a) SIS
b) SIR
c) Both SIS and SIR
d) None of these

(28) A stone is just released from the window of a train moving along a horizontal straight track. The stone will hit the ground following

a) Hyperbolic path
b) Straight path
c) Circular path
d) Parabolic path

(29) A stone, thrown up is caught by the thrower after 6s. How much high did it go? g = 9.8 m/s^2

a) 44.1 m
b) 44.0 m
c) 44.3 m
d) 44.2 m

(30) Two stones of different masses are dropped simultaneously from the top of a building

a) Smaller stone hit the ground earlier

b) Larger stone hit the ground earlier

c) Both stones reach the ground simultaneously

d) Which of the stones reach the ground earlier depends on the composition of the stone

(31) The equation of the curve in which the tangent at a point is always perpendicular to the line joining the point to the origin is

a) $(x-\alpha)^2+(y-\beta)^2=a^2$

b) $x^2+y^2=a^2$

c) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

d) $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

(32) Example predator-prey relationship?

a) A lion eating a zebra

b) A zebra eating grass

c) A human eating fruit

d) None of these

(33) In Domar Macro Model, if $S(t), I(t), Y(t)$ are the Savings, Investment and National Income respectively at time t then the assumption 'Savings are proportional to national income' is represented by

a) $S(t)=\alpha Y(t),\ \alpha>0$

b) $I(t)=\beta Y'(t),\ \beta>0$

c) $S(t)=I(t)$

d) $I(t)=\beta Y(t),\ \beta>0$

(34) In Domar Macro Model, if $S(t), I(t), Y(t)$ are the Savings, Investment and National Income respectively at time t then the assumption 'All savings are invested' is represented by

a) $S(t)=\alpha Y(t),\ \alpha>0$

b) $I(t)=\beta Y'(t),\ \beta>0$

c) $S(t)=I(t)$

d) $I(t)=\beta Y(t),\ \beta>0$

(35) The solution of the motion of a projectile represents

a) a circle

b) a straight line

c) an ellipse

d) a parabola

(36) A particle of mass m is projected from the origin in vacuum with velocity V inclined at an angle α to the horizontal. Then the range of the particle is

a) $\frac{V^2\sin 2\alpha}{g}$

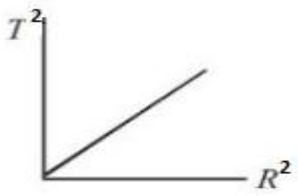
b) $\frac{V\sin 2\alpha}{g}$

c) $\frac{V^2\cos 2\alpha}{g}$

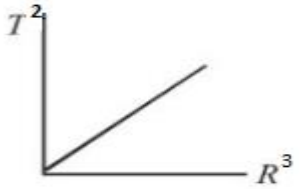
d) $\frac{V^2\cos\alpha}{g}$

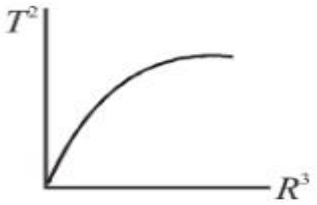
(37) Which of the following graph represents the time period of the planet moving around the sun? [R= semimajor axis of the path]

a)



b)

c) 

d) 

(38) A particle is moving under central force about a fixed centre of force. Choose the correct Statement

a) The motion of particle is always on a circular path.

b) Its angular momentum is conserved

c) Its kinetic energy remains constant

d) None of these

(39) For parabolic orbit the values of energy E and eccentricity ϵ are _____

a) E=0 and ϵ=1

b) E>0 and ϵ>1

c) E>0 and ϵ=1

d) E>0 and ϵ=0

(40) For circular orbit the value of eccentricity _____

a) ϵ>1

b) ϵ=1

c) ϵ<1

d) ϵ=0

(41) From Kepler's law of orbit, we can infer that the sun is located _____ of the planet's orbit.

a) at the centre

b) at one of the foci

c) at both foci

d) anywhere along the semi-minor axis

(42) The orthogonal trajectories of $\frac{x^2}{a^2+\lambda}+\frac{y^2}{b^2+\lambda}=1$ is

a) $(xp+y)(x-py)=p(a^2-b^2)$

b) $(xp-y)(x+py)=p(a^2-b^2)$

c) $(xp+y)(x-py)=p(a^2+b^2)$

d) $(xp-y)(x+py)=p(a^2+b^2)$

(43) The orthogonal trajectories of the family $r=2a\cos\theta$ is

a) $r=2b\cos\theta$

b) $r=2b\sin\theta$

c) $r=2b\cot\theta$

d) $r=2b\tan\theta$

(44) The projectile attains the maximum height $\frac{V^2\sin^2\alpha}{2g}$ at time

a) $\frac{V\sin\alpha}{g}$

b) $\frac{V\cos\alpha}{g}$

c) $\frac{V\sin^2\alpha}{g}$

d) $\frac{V\cos^2\alpha}{g}$

(45) The mathematical model of the curves for which tangent makes constant angle with radius vector is represented by the differential equation

a) $r\frac{dr}{d\theta}=\tan\alpha$

b) $r\frac{d\theta}{dr}=\tan\alpha$

c) $\frac{dr}{d\theta}=\tan\alpha$

d) $\frac{d\theta}{dr}=\tan\alpha$

(46) For the irrotational flow

a) $\bar{V}=\text{grad}\Phi$

b) $\bar{V}=-\text{grad}\Phi$

c) $\vec{V} = \text{curl}\vec{\Phi}$

d) $\vec{V} = -\text{curl}\vec{\Phi}$

(47) For an incompressible fluid, the equation of continuity is

a) $\frac{\partial \rho}{\partial t} + div\left(\rho \vec{V}\right) = 0$

b) $\frac{\partial \rho}{\partial t} = div\left(\rho \vec{V}\right)$

c) $div\left(\vec{V}\right) = 0$

d) None of these

(48) If $\rho = 0$, then the Poisson's equation, $\text{div}(\text{grad}\Phi) = -4\pi\rho$ becomes

a) Wave equation

b) Heat equation

c) Laplace equation

d) None of these

(49) The Euler-Lagrange equation of calculus of variations is in the form

a) $\frac{\partial F}{\partial u} - \frac{\partial}{\partial x}\left(\frac{\partial F}{\partial u_x}\right) - \frac{\partial}{\partial y}\left(\frac{\partial F}{\partial u_y}\right) = 0$

b) $\frac{\partial F}{\partial u} + \frac{\partial}{\partial x}\left(\frac{\partial F}{\partial u_x}\right) + \frac{\partial}{\partial y}\left(\frac{\partial F}{\partial u_y}\right) = 0$

c) $\frac{\partial^2 F}{\partial u^2} - \frac{\partial}{\partial x}\left(\frac{\partial F}{\partial u_x}\right) - \frac{\partial}{\partial y}\left(\frac{\partial F}{\partial u_y}\right) = 0$

d) None of these

(50) Which among these is used to specify a particular problem which we consider for solving in CFD?

a) Boundary conditions

b) Governing equations

c) Governing laws

d) Solution method

(51) The PDE $R(x,y)\frac{\partial^2 z}{\partial x^2} + S(x,y)\frac{\partial^2 z}{\partial x \partial y} + T(x,y)\frac{\partial^2 z}{\partial y^2} + f\left(x, y, z, \frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}\right) = 0$ can be transformed to $\frac{\partial^2 \zeta}{\partial \xi \partial \eta} = \Phi\left(\xi, \eta, \zeta, \zeta_\xi, \zeta_\eta\right)$ if

a) $S^2 - 4RT > 0$

b) $S^2 - 4RT = 0$

c) $S^2 - 4RT < 0$

d) None of these

(52) The classification of PDEs are governed by _____

a) Their highest order derivatives

b) Their least order derivatives

c) The number of terms

d) The constants

(53) Which of these is not a type of flows based on their mathematical behaviour?

a) Circular

b) Elliptic

c) Parabolic

d) Hyperbolic

(54) These are essential for solving partial differential equations.

a) Boundary conditions

b) Physical principle

c) Mathematical model

d) Algebraic equations

(55) Where do we encounter partial differential equations in CFD?

a) Physical models

b) Assumptions

c) Governing equations

d) Discretized equations

(56) D'Alembert's solution of one dimensional wave equation is in the form

a) $u = f(x+ct)+g(x-ct)$

b) $u = f(x+ct)+g(x+ct)$

c) $u = f(x-ct)+g(x-ct)$

d) All of these

(57) The partial differential equation of a vibrating membrane in three dimension is

a) $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=0$

b) $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=\frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$

c) $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=\frac{1}{c^2}\frac{\partial u}{\partial t}$

d) $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=\frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$

(58) Let T be the tension of the elastic string held tightly between the points A & B corresponding to x=0 nad x=L. If the density of the string material is ρ, the equation of wave in that string is

a) $\frac{\partial^2 u}{\partial x^2}=\frac{\rho}{T}\frac{\partial^2 u}{\partial t^2}$

b) $\frac{\partial^2 u}{\partial x^2}+\frac{\rho}{T}\frac{\partial^2 u}{\partial t^2}=0$

c) $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial t^2}=\frac{\rho}{T}$

d) $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial t^2}=0$

(59) Pick the region in which the following differential equation is hyperbolic.

$$yu_{xx}+2xyu_{xy}+xu_{yy}=u_x+u_y$$

a) $xy \neq 1$

b) $xy \neq 0$

c) $xy > 1$

d) $xy > 0$

(60) Which of the following is elliptic?

a) Laplace equation

b) Wave equation

c) Heat equation

d) $u_{xx}+2u_{xy}-4u_{yy}=0$